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MatLab Bvp4c Technique to Compute Thermophoresis and Brownian Motion in Nanofluid Flow Over a Transient Stretching Sheet

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This physical phenomenon examined the transport mechanisms of heat and mass within a nanofluid thin film. The nanofluid thin film is situated over an unsteady stretching sheet, which is one of the pioneering contributions to the field, focusing specifically on the flow dynamics of nanofluid thin films. This foundational framework is established by Buongiorno's fluid model. The mathematical model is applied for the evaluation of the nanofluid film, which adeptly weaves in significant phenomena, including Brownian motion as well as thermophoresis. The mathematical model is achieved in the form of non-linear partial differential equations (PDEs) for computation with the help of computer applications. Firstly, the analytical framework of similarity transformations is applied to non-linear PDEs to convert them into ordinary differential equations (ODEs). Secondly, these ODEs have been critically examined and prepared for coding in MatLab by reducing their high order into first order. The software Mathematica and MatLab have been employed to solve the boundary value problem (BVP). The built-in BVP4c solver is applied to obtain accurate solutions in the form of graphs and numerical values. The current analysis yields significant results revealing that both the free surface temperature and the volume fraction of nanoparticles tend to increase in response to variations in both unsteady conditions and magnetic parameters. Furthermore, the outcomes demonstrate that the interaction among diverse nanofluid variables with the phenomenon of viscous energy loss contributes to a reduction in the overall heat transfer rate. The potential effect of these proficient thermal management techniques is crucial, especially in microelectronics and energy systems.
Keywords: Bvp4c approach, Lobato-IIIA, Computer Applications, MatLab Solver, Mathematica NDSolve, Nano fluid, Heat transfer, Boundary layer, Unsteady Stretching sheet, Mass transfer

Nomenclature:			
T	Temperature of fluid	u, v	Velocity components
Ec	Eckert number	h(t)	Film thickness
T_w	Sheet temperature	C_w	Volume fraction of nanoparticles in the sheet
D_B	Brownian diffusion coefficient	C	Volume fraction of the nanoparticle
N_t	Thermophoresis Parameter	D_T	Thermophoresis diffusion coefficient
N_b	Brownian motion parameter	Nu	Nusselt number
Re_x	Local Reynold number	p	Pressure
K	Thermal conductivity	Pr	Prandtl number
M	Hartmann number	b	Positive constant
T	Temperature	Sc	Schmidt number
Sh_x	Local Sherwood number	φ_a	nanoparticle volume fraction
Abbreviations.			
BNF	Buongiorno nano fluid	FDC	Finite Difference Collocation

Introduction:

The study of the unsteady motion of a stretching sheet is one of the pioneering contributions to the field, focusing specifically on the flow dynamics of nanofluid thin films. The Buongiorno's fluid model generates a combination of two effects: Brownian motion and thermophoresis. The Brownian motion characterizes the chaotic movement of particles, and thermophoresis indicates particle displacement driven by temperature variations. Nanoparticles in any fluid, viscous or inviscid, improve the properties of the fluid. These properties may consist of thermal conductivity, heat transfer, and mass transfer. This behavior of the flow of nanoparticles indicates a complex interplay between the free surface and volume fraction. Thus, this examination contributes meaningfully to the prevailing knowledge concerning nanofluids while also initiating paths for upcoming inquiries into the enhancement of heat transfer functions in advanced materials. The use of nanofluids in many fields of engineering is widely common such as cooling systems when better thermal conductivity is provided, heat exchangers when it transfers from one medium to another medium that is from heat to cool liquids without mixing them, radiators when it plays its role to cool the engine by antifreezing mixture through its fins, and nuclear reactors when extraction of geothermal is captured by producing electricity. Since the nanoparticles possess a high rate of friction, they produce an enormous change in heat and mass transfer within the fluid. The researchers[1],[2],[3] investigated the process of heat and mass transfer on a simple base fluid on different fluid models with different geometries. They did not consider the nanofluid flow in these physical problems, which is why the behavior of heat and mass transfer with fluid flow is quite different as compared to[4], where the heat transfer on a mixed convective nanofluid flow of Walter-B has been discussed. In the industries of coating, such as wire and fiber coating, liquid film flows are primarily used. For minimal friction, strength, performance, and beauty, the optimal rate of heat and mass transfer is necessary. Due to the low thermal conductivity, common fluids, including water, mineral oils, and ethylene glycol, have poor convective heat transfer. The production of the related heat transfer procedure rises when the fluid's thermal conductivity increases. In light of these facts, several techniques have been created to enhance the thermal conductivity of normal fluids. In the recent era, the fluid-based nanoparticles have drawn the attention of researchers. In this context, [5] investigated the activation energy and magnetohydrodynamics with Dufour and Soret effects in rotating flow of different nanofluid models. They explored that their work best suits applications with the food industry, biomedical, industries relevant to energy systems, and technologies relevant to aerospace systems due to temperature distribution, thermophoresis, Brownian motion effects, and heat source. A uniform physical model for the nature of transient thin-film flow of a Maxwell nanofluid across a spinning disc has been examined by[6] when a non-linear thermal radiation and a uniformity were present.[7] Observed during exploring the properties of heat transport in finite film flow of human blood using carbon nanotube (CNT) nanofluids across a stretchy vertical cylinder. Human blood is utilized as a basis liquid for two types of carbon nanotube nanoparticles: that is, single-walled carbon nanotubes and multi-walled carbon nanotubes.

Nanoscience is an exceptional method of altering a working fluid's characteristics. From an industrial and technological perspective, the qualities of heat transfer must be taken into consideration when nanofluids are flowing. Continuing the work,[8] looked at the study investigations of the three-dimensional mobility of thin-film nanomaterials on a stretchy rotating inclined surface. The mass and heat transfer process of MHD unsteady thin film flow of an aluminum–copper/water hybrid nanofluid driven by radiation, thermophoresis, and Brownian motion has been examined by[9]. [10] examined the Darcy Forchheimer two-dimensional thin film fluid of nanoliquid in this article. They observed that nanoliquid flows through a stretchy, flat, unstable, and unsteady sheet. Carbon nanotubes, or CNTs, are a type of nanomaterial found in nanoliquids.[11] have studied heat transfer and pseudo-plastic magnetohydrodynamic (MHD) nanofluid flow in a finite film over an internally heated unstable stretching surface. Four distinct kinds of nanoparticles, *TiO2, Al2O3, Cu, and CuO,* are taken into consideration, and the base fluids are pseudo-plastic carboxy methyl cellulose and water. According to[12], a viscous nanofluid thin-film flow over a rotating horizontal disc has presented a thermal nonlinear radiation with the impact of MHD. The heat transmission of a thin film flow in nanofluids through an unstable stretching surface is evaluated by[13]. Using water as the base fluid, three distinct kinds of nanoparticles have been considered: *copper Cu, alumina Al2 O3, and titania TiO2.*[14] explained the impact of nanofluids on the stretched evaporating semilunar cartilage in a microchannel, a thin film evaporation model has been created using kinetic theories and the augmented Young-Laplace equation. The structural disjoining pressure, a slim porous coating layer at the boundary created from deposition of nanoparticles, and changes in thermophoresis properties in comparison to the base fluid are some of the consequences of the nanofluid.

Because the inclusion of nanoparticles in fluids significantly increases the fluid's thermal conductivity and, consequently, improves the properties of heat transfer,[15] investigation of nanofluids has become a popular issue among researchers. The objective of the study[15] is to investigate the finite unstable thin-film flow of upper convected Maxwell fluid caused by a disc horizontally rotating when nanoparticles are present. The contribution of a study by[16] examined the Casson fluid's liquid film flow across a stretched sheet of blood-based hybrid nanofluid with carbon nanotubes of different viscosities. The endeavour[16] is particularly beneficial because of the several uses for carbon nanotubes (CNTs), which include excellent thermal and electrical conductivity, high tensile strength, 18% more elasticity than other widely used nanoparticles, low thermal expansion coefficient, and better electron discharge. A magnetic field that is normal to the field of flow is employed in this concept. By combining the effects of thermal radiation and couple stress,[17] discussed the heat rate of energy transfer in a thin film within 3D nanofluid (water-based) flow across a spinning surface.

The Buongiorno fluid model contains suspended nanoparticles in which the motion and heat transfer of nanofluids are described. Buongiorno fluid is a two-phase model that describes the motion of the base fluid as well as nanoparticles. The mechanism of motion of particles due to temperature gradients is called thermophoresis in the Buongiorno model, and the other motion of particles, which is due to molecular collisions, is called Brownian diffusion. As this model enhances thermal conductivity and heat transfer rate, the energy systems and cooling technologies are usually dealt with by this fluid model. The impacts of viscous dissipation and interior heat generation on the Buongiorno fluid model flow are often considered to analyze the combined effects of thermal transport and energy conversion in boundary layer flows.. This mathematical model predicts the improvement of the thermal efficiency of fluids due to nanoparticles. The effects of thermophoresis flow, Brownian motion, mass, and heat transfer from a flat plate with a defined surface of heat flux are examined using the Buongiorno model. [18] talked about the speed of mathematical frameworks used to study the behavior of fluids that have better heat transfer and thermal conductivity coefficients than their base fluid. These improvements go beyond the typical thermal-conductivity effect and are not in line with the predictions of conventional pure-fluid correlations. The natural convection heat transfer of nanofluid in a two-dimensional square cavity with multiple heater-cooler pairs was covered by[19]. The Buongiorno fluid model can be extended to MHD effects in which the nanoparticles are electrically conducting nanofluids when magnetic fields are present. The mass and heat transfer properties of unstable nanofluid flow between parallel plates were examined by[20]. Continuing the process of research,[21] the Buongiorno model was applied to systematically investigate the flow of nanofluids through a stretching sheet. The thermophysical features of nanoliquids were studied as well. Additionally, the Brownian motion and thermodynamic motions characteristics brought on by the nanofluid are represented by the Buongiorno model. Using Buongiorno's model,[22] have examined the weight and heat transfer of the nanofluid thin layer across an unstable stretching sheet above a transient extending sheet with flow behavior generated. The motion, temperature, and transfer of mass within both divergence and convergent channels in the presence of a magnetic field were examined by[23]. It is also claimed that the walls of the canal are growing or shrinking. Buongiorno's model is used to pose the challenge for nanofluids. The nanofluid model incorporates the significant impacts of thermophoresis and Brownian motions. [24] analyzed numerically Buongiorno nanofluid model with electrically conducting magnetic field, viscous dissipation, and heat-generating flow through a wedge. They perceived that with the inclusion of the parameter of Brownian motion, the temperature profile is enhanced. [25] described a Buongiorno nanofluid flow with the effect of MHD through an infinite rotating disk and observed that the negative sign with axial velocity is caused by the downward fluid motion due to the rotation of the disk. The special case of his study can also be obtained by the analysis of a no-slip boundary. The flow of heat transfer and boundary layer characteristics via a porous isothermal surface that is stretching or contracting was investigated by[26] using nanofluid and hybrid nanofluid flows. This hybrid flow, which additionally analyzes the Buongiorno nanofluid model, is known as the modified Buongiorno nonliquid model. The effects of physical parameters on the temperature function, velocity function, and concentration function were examined by[27] and were also shown in graph form.

Further,[28] put forward Buongiorno's fluid model for viscoelastic flow with magnetohydrodynamics effects through a lubricated heated surface. It is analyzed during the study that when the Prandtl number starts to increase, the values of the Nusselt number increase in the model. [29] advanced the Buongiorno model towards the nanofluid flow through a porous medium on a linear shrinking sheet. They examined the dual nature of Darcy and Forchheimer law using Buongiorno's model. The viscoelastic property of the Buongiorno model is also a novel approach to examining the fluid flow for the production processes. [30] obtained a radiative heat transfer mechanism for viscoelastic Buongiorno fluid flow through a stretching sheet. In this study, the Buongiorno fluid model over an unsteady stretching sheet has been overlooked to get solutions to the problem by using computer applications. None of the above-discussed Buongiorno fluid models has been analyzed by the BVP4c technique. The mathematical boundary value problems for ODEs are solved by BVP4c in MatLab, and graphical pictures are obtained by the MatLab software. The mathematical package of the finite difference Collocation method has been used in BVP4c to compute numeric values and graphs. The governing equations for the Buongiorno fluid model are nonlinear and are often solved using numerical methods. Moreover, it is a boundary layer flow over stretching sheets, flat plates, and thin films, etc. Following the introduction and abstract, the provided study is structured as follows in Section 1: Using Buongiorno's approach, Section 2 is ready for mathematical modeling on nanofluid unstable thin films. The solution mechanism for thermophoresis and Brownian motion in Buongiorno's model is built in Section 3. Section 4 presents the whole process of the solution with a flow chart. Results and Discussions are shown in Section 5, solution charts are provided in Section 6, and the study's conclusion is presented in the last section.

The objectives of this study are to offer a realistic design for nanofluid transport that classical fluid models ignore. The Buongiorno fluid model is basically an understanding of the unique heat and mass transfer characteristics of nanofluids that cannot be described by classical single-phase models. Brownian motion and thermophoresis are really influencing mechanisms of nanoparticle transport in nanofluids.

Mathematical Model of BNFs:

Consider the nanofluid in a thin film through an unsteady stretching sheet with the following geometrical properties[22]. The transient sheet can be extended at a linear velocity

$$U_w = \frac{bx}{1-\gamma t},$$

where t is for time, γ and b are the positive values of constants, where x denotes the abscissa originating from point O (refer to figure 1). Considering a homogenous magnetic field of intensity with a small magnetic Reynolds number, the impact of an externally induced magnetic field is said to be ignored.

The magnetic field is oriented in the positive y-direction, which is perpendicular to the plane.

$$\text{Here } T_w(x,t) = T_0 - T_r\left(\frac{bx^2}{2\nu}\right)(1-\gamma t)^{-\frac{3}{2}},$$

the surface temperature distribution and the nanoparticle volume fraction

$$C_w(x,t) = C_0 - C_r\left(\frac{bx^2}{2\nu}\right)(1-\gamma t)^{-\frac{3}{2}}$$

are postulated to change with the distance x from the slit, C_0 and T_0 indicates the volume fraction of nanoparticle and temperature of the nanofluid at the slit, whereas C_r and T_r denote the constant reference nanoparticle volume fraction and the constant reference temperature, respectively, such that $(0 < T_r < T_0 \ ; 0 < C_r < C_0)$.

$B(t)$ is a magnetic field in a transverse direction.

Given as:

$$B(t) = B_0(1-\gamma t)^{-\frac{1}{2}},$$

A thermal equilibrium state is assumed initially for suspended nanoparticles and the base fluid.

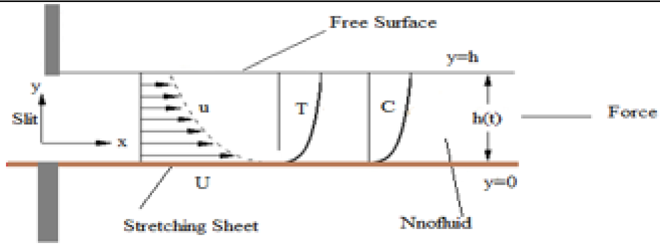


Figure 1. A visual representation of the problem

The governing equations of conservation of mass, momentum, energy, and nanoparticles fraction[19],[22] are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \quad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2(t)}{\rho_f}u, \quad (2)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{c_\rho}(\frac{\partial u}{\partial y})^2+\tau[D_B(\frac{\partial C}{\partial y}\frac{\partial T}{\partial y})+(\frac{D_T}{T_\infty})(\frac{\partial T}{\partial y})^2], \quad (3)$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B(\frac{\partial^2 C}{\partial y^2})+(\frac{D_T}{T_\infty})(\frac{\partial^2 T}{\partial y^2}), \quad (4)$$

Boundary conditions[22] are

$$u=U_w,\quad v=0,\quad T=T_w,\quad C=C_w,\text{as }\ y=0,$$

$$\frac{\partial u}{\partial y}=\frac{\partial T}{\partial y}=\frac{\partial C}{\partial y}=0,\quad v=\frac{\partial h}{\partial t}\quad \text{as }\ y=h(t). \quad (5)$$

Methodological Approach:

The mathematical model of the problem given in Section 2 is in the form of non-linear PDEs. (1) – (5). The step-by-step methods of this study are given below

Convert the above PDEs with boundary conditions (1) – (5) into nonlinear ODEs with the help of the following similarity variables:

$$\psi=(\nu_f x U_\infty)^{\frac{1}{2}} f(\eta),\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\quad \eta=(\frac{U_\infty}{\nu_f x})^{\frac{1}{2}}y,\quad \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}, \quad (6)$$

After applying the similarity transformations given in Eq. (6), the whole mathematical model with boundary conditions from Eq. (1) to eq. (5) is adapted in the form of nonlinear high-order ODEs. These high-order ODEs have been made able to be fed into a computer to get a solution. That is why the first order ODEs are achieved from this high order by assigning values to them. Now the system of first-order ODEs with boundary conditions is ready to code for MatLab BVP4c. The finite difference Collocation method is used to solve the system of ODEs at the backend of MatLab BVP4c. The accuracy of the FDC method is very high compared to other methods for the solution of a system of ODEs. The numeric values are arranged, and graphs are plotted using these numeric values.

Solution Process:

Eqs. (1) to (5) yield the following non-dimensional system of nonlinear ODEs by using the similarity variables given in Eq. (6) as:

$$f'''+ff''-f'^2-S(f'+\frac{\eta}{2}f'')-Mf'=0, \quad (7)$$

$$\frac{1}{\Pr}\theta''+f\theta'-2f'\theta-\frac{S}{2}(3\theta+\eta\theta')+Ecf''^2+Nb\phi'\theta'+Nt\theta'^2=0, \quad (8)$$

$$\phi''+Sc(f\phi'-2f'\phi-\frac{S}{2}(3\phi+\eta\phi'))+\frac{Nt}{Nb}\theta''=0, \quad (9)$$

Hence, boundary conditions are

$$f(0)=0, f'(0)=1, \theta(0)=\phi(0)=1, f(\beta)=\frac{S\beta}{2}, f''(\beta)=0, \theta'(\beta)=\phi'(\beta)=0, \quad (10)$$

The expressions representing various physical quantities can be formulated as.

$$\Pr=\frac{\nu}{\alpha},\ Sc=\frac{\nu}{D_B},\ Nb=\frac{\tau D_B(C_w-C_\infty)}{\nu},\ Nt=\frac{\tau D_T(T_w-T_\infty)}{T_\infty \nu},\ Nu=\frac{xq_w}{k(T_w-T_\infty)},$$

$$Sh=\frac{xq_m}{D_B(C_w-C_0)},\ C_{fx}=\frac{\tau_w}{\frac{1}{2}\rho U_w^2},\ B(t)=B_0(1-\gamma t)^{-\frac{1}{2}}, \quad (11)$$

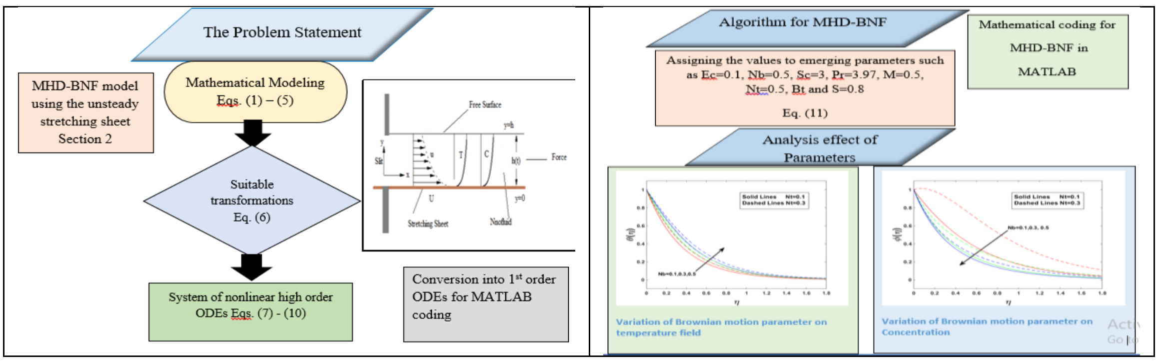


Figure 2. Flow Chart of Work

Results and Discussion:

The BNF model is considered with the properties of unsteadiness of the shrinking sheet and the magnetic field. The mathematical model was reviewed, and the nonlinear set of governing equations was obtained. The results of the BNF model described in Equations (07) – (10) are examined using the BVP4c MatLab implementation. This study presents three distinct BNF model scenarios with respect to the temperature, velocity, and concentration profiles, as shown in graphs of Fig. 3 – Fig. 6. The effects of emerging parameters such as Eckert number, thermophoresis motion, Brownian motion, Schmidth number, Prandtl number, unsteadiness parameter and magnetic parameter: S, M respectively are visually demonstrated in present research. Figure 3 shows the primary velocity profile, temperature profile, and nanoparticle volume fraction for the two different unsteadiness parameters and magnetic parameter values that are varied. As illustrated in Fig. 3a, the dimensionless axial velocity is found to be larger in the hydrodynamic boundary layer and decreases as the magnetic field and unsteadiness parameter rise. In fact, physically, the magnetic field generates Lorentz forces that compete against the motion, and hence, the main velocity decreases. It is noticed that the magnetic field strongly represses the velocity in directions perpendicular to its lines of force. Magnetic and unsteadiness parameters damp the turbulence effectively and bring a laminar-type flow. The numerical solution shows that the free stream velocity attains a certain thickness of thin film. For every low value of parameter S, the film thickness also lowers. Consequently, as magnetic and unsteadiness parameters are increased, the thickness of the boundary layer decreases.

Furthermore, as the dimensionless temperature rises, the convergence process is methodically investigated for various parameters, including M and S, as illustrated in Fig. 3b.

The temperature profile causes the internal thermal boundary layer to decrease, which causes the dimensionless film to begin to thin. When a magnetic field and an unsteadiness parameter are present, the dimensionless heat transfer increases, which causes the thermal boundary layer to thicken. Figure 3c depicts the thinning of the nano-fluid film in the presence of both M and S parameters. The effects of the magnetic field and unsteadiness parameter are noticeable for nanoparticle volume fractions.

When all parameters remain constant except the thermophores parameter fluctuates, the effect of the Brownian motion parameter is shown in Fig. 4. It is clear that when the Nb value increases, the temperature profile progressively rises while the concentration profile of nanoparticles gradually falls. This pattern is clearly depicted in Fig. 4a, where the size of the border layer increases with increasing Nb. Furthermore, the concentration profile for various Nb values under the same conditions, that is, the values of Ec = 0.1, Nt=0.5, Sc=3, Pr=3.97, M=0.5, and S=0.8. The concentration profile provides important information on how different Nb values affect. In contrast, Fig. 4b shows that when Nb values rise, the thickness of the boundary layer decreases. By encouraging collisions between nanoparticles and the base fluid, the improvement of Brownian motion demonstrates a crucial role in enhancing the thermal properties of the nanofluid. Nt affects the $\theta(\eta)$ profiles in the following scenarios: Ec=0.1, Nb=0.5, Sc=3, Pr=3.97, M=0.5, and S=0.8 (Fig. 4a). The physical parameters show the variations in the $\theta(\eta)$ pattern when Nt takes on different values graphically. As the thickness of the zero-dimensional film decreases, the thickness of its thermal boundary layer increases, causing the dimensionless temperature to increase with the magnetic field for all values of the unsteadiness parameter. Additionally, both values cause the nanofluid sheet's thickness to decrease. Figure 5 shows how the Schmidt number affected dimensionless concentration and how the Prandtl number affected non-dimensional temperature. In Figure 5a and Figure 5b, the effects of newly discovered factors on the temperature without dimensions and nanoparticles volume fraction for four distinct fluids are displayed, respectively. The thermal boundary layer thins because of the fluid's steady decrease in thermal conductivity, which is indicated by a high Prandtl number. As seen in Fig. 5a, the film thickness stays constant for a given Prandtl number value. When viscous dissipation is absent, the thickness of the thermal boundary layer is at its lowest, and viscous dissipation is observed to progressively rise. The effect of the Schmidt number on the volume fraction of nanoparticles in dimensionless form is shown in Figure 5b. It is demonstrated that when the Sherwood number rises, the thickness of the boundary layer decreases. This procedure locates the fluid flow where mass diffusion, convection, and instantaneous momentum processes take place. It assesses the mass transfer boundary layer and hydrodynamic film thickness physically. Notably, for high Schmidt numbers, the effects of viscous dissipation on the volume fraction of nanoparticles are essentially insignificant. The impact of the magnetic field and unsteadiness parameter on the volume fraction of dimensionless nanoparticles is shown in Fig. 6. The $\theta(\eta)$ profile somewhat improves with an increase in thermophores Nt. Additionally, the thickness of the boundary layers grows as Nt rises, which indicates a stronger effect on the nanofluid flow.

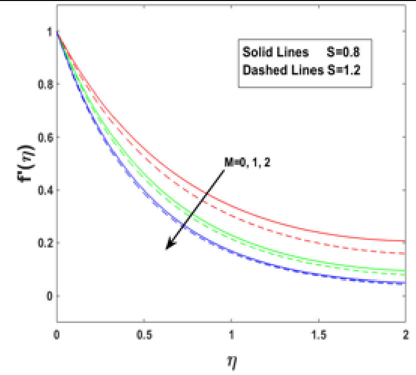


Figure 3a Variation of M for f'(η).

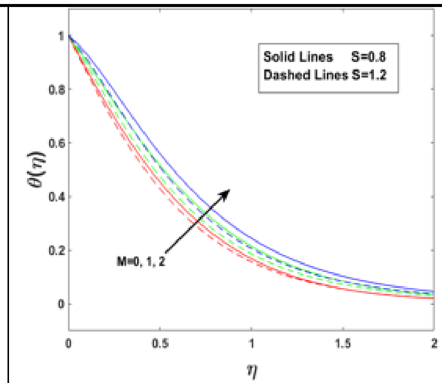


Figure 3b Variation of M for θ(η).

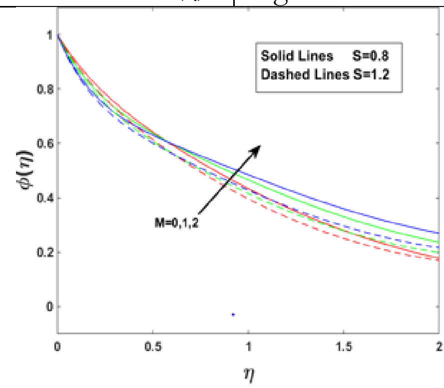


Figure 3c Variation of M for ϕ(η).

Figure 3. Effects of Magnetic Parameter on Main velocity, Temperature field, and Concentration

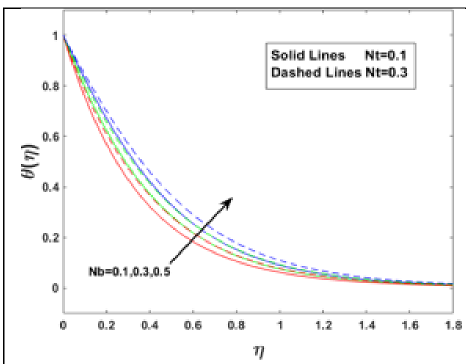


Figure 4a Variation of Nb for θ(η).

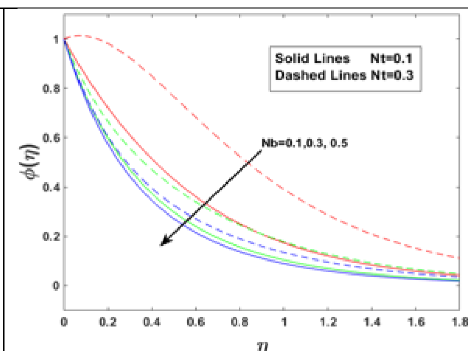


Figure 4b Variation of Nb for ϕ(η).

Figure 4. Effects of Brownian Motion parameter on Temperature field and Concentration

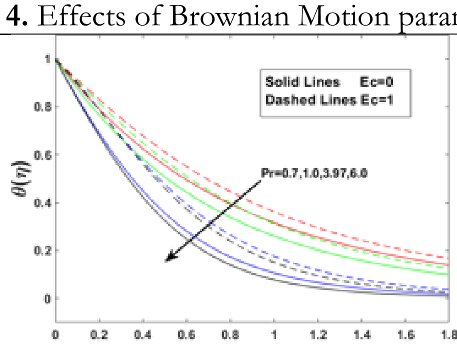


Figure 5a Variation of Pr for θ(η).

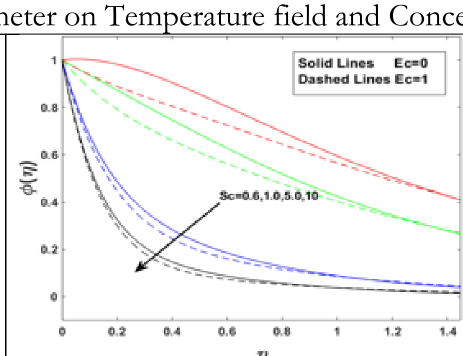
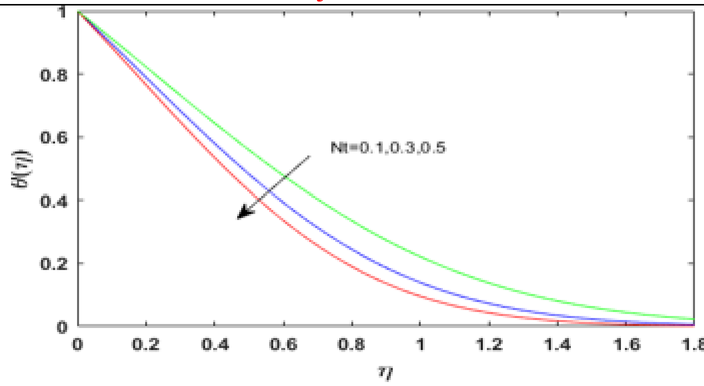


Figure 5b Variation of Sc for ϕ(η).

Figure 5. Effects of Prandtl number on the Temperature field and Schmidt number on Concentration



Solution plot for Energy Profile

Figure 6. Variation of thermophoresis on Temperature Field

The results of the mathematical model[22] are compared directly with those of this study. MATLAB bvp4c solver results of this study are more accurate than the Maple bvp4c solver in[22].

Conclusion:

This work deals with the MATLAB bvp4c solver accuracy compared with the Maple bvp4c implementation. Despite the fact that both programs are made to solve boundary value problems (BVPs) for ordinary differential equations that have been converted into first order, their accuracy of results varies. This difference is due to the mathematical numeric methods used in the packages of both programs. MATLAB bvp4c has been designed using the packages of adaptive mesh refinement and collocation method with a 4th-order accurate scheme. Whereas Maple bvp4c has been designed as an approximation of MATLAB's solver, but both have different strategies for coding the system. In order to examine the scenario of solution flow, we have examined the physical problem of Buongiorno's fluid model for heat and mass transmission in a nanofluid layer over an unstable stretched sheet. Two computer programs, MatLab and Mathematica, have been used to study the effects of film thickness, unsteadiness, magnetic field, viscous dissipation, and characteristics of nanofluids on fluid main flow velocity, heat, and mass transfer rates. The main conclusions are:

The free stream velocity decreases as the magnetic and unsteadiness parameters increase.

The increase of both unsteadiness and magnetic factors causes the thickness of the film to rise.

The thickness of the thermal boundary layer is increased by viscous dissipation.

Both unsteadiness and magnetic parameters cause a rise in the free surface temperature and the volume fraction of nanoparticles.

Heat and mass transfer rates rise with the decrease of unsteadiness values.

With both nanofluids, the dimensionless heat transfer rates drop.

BVP4c-MatLab software platform gives high numerical accuracy rather than the BVP-Maple software[22].

It is observed clearly that all the results attained by BVP4c-Matlab in this study have a good match with the results obtained through the BVP-Maple solver in the literature[22].

It is found that MatLab BVP4c tool optimizes highly towards engineering applications and provides the best accuracy with adaptive modification, whereas the Maple BVP solver is good for symbolic and well-defined problems.

The conclusion summarizes the key findings of the range of numerical values attained by BVP4c and highlights the advantages of these numerical values for use in artificial neural networks to design the artificial intelligence environment for the predictions. The conclusion highlights a wide range of effective numeric data that can be used for advancements in the field of artificial intelligence.

Limitations & Future work:

In future work, this article can be extended to the design of artificial Neural Networks. The algorithms and graphical representations of errors, histograms, transition states, and error plots can be obtained by the same software, that is, MatLab by nftools. Limitations include the need to train the data and validations.

Statement of generative AI in scientific writing:

No AI tool is used during the preparation of this study. All wording is according to the process of the problem and self-created. Only the services of Grammarly have been used to check the grammar mistakes throughout the article.

Contribution statement:

Atifa Latif: Writing – original draft, Software, Investigation, Conceptualization, Data curation, Formal analysis, Methodology

Saman Iftikhar: Writing – review & editing, Software, Funding acquisition.

Syed Sheraz Asghar: Resources.

Hadia Ali: Visualization, Writing – review & editing.

Declaration of competing interests

All authors declare that they have no competing financial interests or personal relationships that may appear to influence the work in this manuscript.

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