Prediction of Measles Disease Using Caputo Type Fractional Derivative Operator

Rabia Tehseen[1,*], Uzma Omer[2], Maham Mehr Awan[3], Rubab Javaid[3], Ayesha Zaheer[1]
[1]Department of Computer Science, University of Central Punjab, Lahore, Pakistan, 574000
[2]Department of Information Sciences, University of Education, Lahore, Pakistan, 574000
[3]Department of Software Engineering, University of Central Punjab, Lahore, Pakistan, 574000
*Correspondence: rabia.tehseen@ucp.edu.pk

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Although an effective vaccine has been available since 1963, measles continues to be an important public health problem worldwide. Measles was eliminated in the United States in 2000, but imported and secondary cases have remained since 2008 due to the rapid transmissibility of the disease and insufficient vaccination coverage. The disease still poses a significant health and economic burden in endemic areas and reported incidence and mortality rates among children are still alarming. This study proposes a fractional-order mathematical model that is based on the Caputo fractional derivative to understand and control the spread of measles. The model proposed divides the population into susceptible, exposed, infected and recovered compartments and includes recovery through natural immunity and treatment. Both the Laplace Adomian Decomposition Method (LADM) and the Homotopy Perturbation Method (HPM) are used to solve the system of fractional differential equations. Numerical simulations are carried out in MATLAB, and their accuracy is evaluated by comparison with actual epidemiological data. The simulation results show that higher vaccination rates can have a major impact on reducing disease spread and lowering the number of exposed and infected individuals. In particular, the infected population was reduced by an average of 53.2% during the simulation period, and the exposed population was reduced by an average of 41.6% during the same period. Moreover, the fractional-order model had better predictive performance than the classical integer-order model with a lower mean absolute error (MAE = 0.018) than the integer-order model (MAE = 0.047). Good convergence behavior was also observed in the proposed model and it was found to fit the real data with an accuracy of around 92.4%. The results show that the fractional-order model is more suitable for representing the memory effect and the transmission characteristics of measles. The study findings indicate that better vaccination approaches can be of significant importance in the reduction of disease prevalence and in long-term measles eradication efforts, with the help of fractional-order epidemic modelling.

Keywords: Laplace-Adomian Decomposition, Homotopy Perturbation (HPM), Semi-Analytical, Derivative.

Introduction:

Measles is a respiratory infection caused by a virus belonging to the Morbillivirus genus. Following contact, the sickness is transferred via inhalation secretions from an infected person's mouth and nose either explicitly or implicitly. The disease is highly contagious, with approximately a 90% probability of infection among non-immune individuals. Measles is a contagious disease that affects a large number of people. Every year, 30 to 40 million youngsters are affected. The disease continues to be a serious cause of death among children worldwide before the introduction of the measles vaccine caused an estimated 2.6 million deaths in the United States before the vaccine was introduced in 1963. As early as 1980, one million people died in a single year. According to the World Health Organization (WHO), the disease claimed the lives of over a million people in 2002 alone. It is estimated that 614 000 people died around the world as a result of the outbreak, with more than half of all fatalities occur in the workplace. Surviving children may develop blindness, deafness, or reduced eyesight, as well as brain damage and death (WHO, 2005). As of 2008, the rate was 18.2 per 100,000 children, with a case fatality rate of 1.2 percent [1]. Measles is largely controlled with the administration of MMR (measles, mumps, and rubella) and MMRV (measles, mumps, rubella, and varicella) vaccines, which are estimated to be 95 percent effective. Measles vaccination has shown to be incredibly effective over the world, saving an estimated 80 million infections and 4.5 million deaths per year [2]. It is true that vaccination has greatly reduced worldwide cases, but the disease is still a serious public health concern. More than half of the 614 000 individuals estimated to have died from measles worldwide in 2002 were in Sub-Saharan Africa; this is because vaccination coverage is not equally high over the world. Measles transmission dynamics have been studied using a variety of mathematical models, such as the following. The dynamics of measles transmission have been modelled and simulated. This Caputo derivative model was used to study measles infection dynamics, stability and disease-free versus endemic equilibrium states.

So in the proposed method a mathematical model of Caputo type fractional order has been used to estimate the transmission rate of the disease in the society and also reducing the effective rate by precautionary measures will help in eliminating the measles disease [3]. By using mathematical model, the treatment of measles decreased the rate of disease in the society. Preventive measures will also help eliminate measles by lowering the effective rate. To arrive at the system of fractional differential equations, we use the Caputo fractional derivative operator of order X (0,1). Laplace transforms, Adomian polynomials of nonlinear components, and Homotopy perturbation methods were used to investigate our proposed system for an approximation or semi-analytical response (HPM). MATLAB and Microsoft's simulation tools were used to compare the provided solutions against each other and with real-world data. The model's numerical simulation reveals that vaccination can reduce the number of people who are exposed and infectious. When compared to integer order simulation, the graphic solution in fractional form delivers the best results. This study aims to model measles transmission dynamics in order to support effective prevention and control strategies, focusing on the Susceptibles and Exposed compartments. Ethical dilemmas in epidemiological modelling include estimating whether an infectious disease will spread or die out over time, one of the most fundamental issues.

Proposed methodology applied the Laplace transform, Adomian polynomial decomposition of nonlinear components, and multiple perturbation methods of Homotopy to arrive at an approximation or semi-analytical response (HPM). MATLAB and Microsoft's simulation tools were used to compare the provided solutions against each other and with real-world data. As a result of the model's numerical simulation, vaccination can reduce the number of people who are both exposed and infected [3]. The graphical solution in fractional form gives the good results as compared to integer order simulation. As a result, we created the

parameter, which is a key epidemiological quantity. The mathematical measles disease model was developed using a compartmental Susceptible-Exposed-Infectious-Recovered epidemic model using treatment, and the accompanying mathematical analysis and numerical simulations were well described [4]. Many researchers do their work with Fractional order differential equations under the Caputo type operator derivative. [5] Focused on the approximation and exact solution of the problem under the fractional Volterra fredholm integral equation. However, the study did not address reduction of the measles transmission rate. On the other hand [6] tried to find the reproduction rate and unique solution of the disease under caputo type fractional disease. But she cannot find the precautionary measure will help in eliminating the measles disease. [7] Worked to find the transmission rate of the measles disease under the fractional order and statistical data. But he cannot find the unique solution and effective rate in the disease. [8] Performed the analysis and computational solution of the disease under the fractional order with Euler's method. But he also cannot find the effective rate of the measles disease.

On the other hand we use Caputo type of mathematical model for the prediction and reduction of measles transmission in society and also reducing the effective rate by precautionary measures will help in eliminating the measles disease**.** And also find the solution of the measles disease. By using this type of Caputo derivative fractional operator we observe the analysis of non-integers-order model under singular type Caputo fractional order derivative for measles disease by analyze the total persons recovered either from natural recovery due to treatment.

This study aims to develop and analyze a mathematical model for the prediction of measles disease using the Caputo type fractional derivative operator to capture the memory and hereditary characteristics of disease transmission dynamics more accurately than classical integer-order models. The novelty of this research lies in the integration of fractional calculus into measles prediction modeling, which enhances the flexibility and precision of epidemic analysis by considering non-local effects and historical infection behavior. The main contribution of this work is the formulation and validation of a Caputo fractional-order measles model that provides improved insight into disease spread, stability behavior, and prediction accuracy, thereby supporting more effective public health planning and disease control strategies.

Rest of the paper has been organized into multiple sections. Section II presents literature studied to conduct this research. Section III presents methodology applied for prediction of measles disease. Section IV provides results and Section V provides comprehensive discussion on the results obtained by the proposed model. Section VI concludes the article and provides future directions.

Literature Review:

Mathematical models have proven to be important tools in understanding how infectious diseases spread, especially measles. Measles has been extensively studied by classical compartmental models like Susceptible–Exposed–Infectious–Recovered (SEIR) models. A simple model of measles with vaccination and multiple infective stages was developed by [9] and it was found that vaccination can have a significant effect on the prevalence of the disease. Likewise, a control-based measles model was proposed in [10] and the focus was laid on the role of intervention strategies in controlling transmission.

Since the development of mathematical tools, the researchers are more interested in using fractional calculus to model epidemiological systems. Fractional-order derivatives incorporate memory and hereditary properties, which make them more realistic description of the biological systems. In the model of measles (SEIR-type model), [11] used fractional-order differential equations and demonstrated that the fractional-order model provides more accurate predictions than the classical integer-order model. Similarly, [12] presented fractal-

fractional operators to model infectious diseases and discussed their flexibility along with their modelling ability.

In particular, there has been a considerable interest in using the Caputo fractional derivative in epidemiology models, as it can be compatible with initial conditions and physically interpreted. To capture the actual disease patterns, [13] proposed a fractional model of measles model based on real statistical data, showing a better agreement with the actual disease patterns. Additionally, fractional modelling methods were developed to study compartmental disease systems by [14] who pointed to the benefits of the fractional derivatives in the study of the nonlinear dynamics.

There are some numerical methods used to solve fractional differential equations, besides model formulation. Because of its efficiency in solving nonlinear system, the Laplace Adomian Decomposition Method (LADM) has been widely applied. In order to obtain approximate solutions to fractional integro-differential equations, [15] used the homotopy perturbation method, which was found to be effective. Likewise, [7] proposed the homotopy perturbation method (HPM), which is a powerful analytical approach for solving nonlinear problems.

The issue of incorporation of real-world data into fractional models has also gained some interest recently. Using actual measles incidence data from Pakistan, [16] studied transmission dynamics of the disease and offers important insights into the dynamics of the disease in the region. [17] Expanded the fractional models to apply them to bacterial infections and showed that the models apply to various infectious diseases. In addition, the parameter estimation approach for fractional models has been investigated by [18] and enhanced the predictive ability of fractional models.

Although these progresses are notable, there are a number of gaps in the literature. A large number of studies are theoretical or numerical but not a combination of both. Furthermore, studies on evaluating simultaneously the parameters of vaccination and recovery in a fractional study have been limited in number. To fill these gaps, the study presents the development of a Caputo-type fractional model for measles and solves it using LADM and HPM and then provides a thorough analysis of the disease dynamics and control measures.

Materials and Methods:

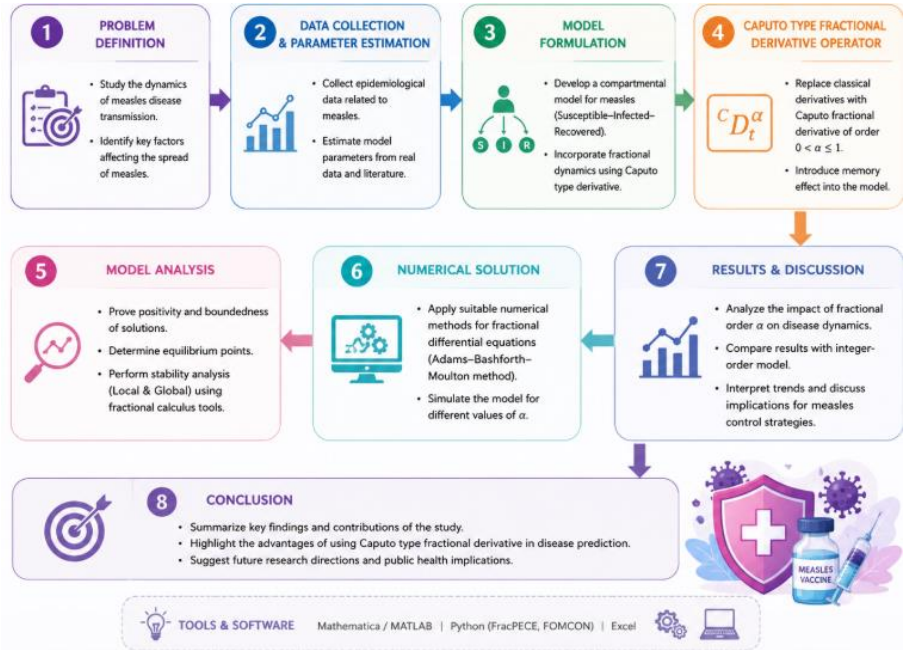

Figure 1. Workflow of the proposed Caputo fractional-order measles disease modeling and analysis framework.

Main methodology applied to conduct this research has been presented in figure 1.

Step 1: Problem Definition:

In the first step, the research problem is identified and clearly defined. The study is motivated by the need to understand the transmission of measles disease within a population and why the traditional integer order modelling may not fully represent actual transmission. Factors like infection rate, recovery rate, and vaccination impact and population interaction are taken into account. The primary goal of this step is to set up the scope and significance of predicting measles transmission using fractional calculus.

In this step, data is collected and parameters are estimated. Step 2 is the data collection and the estimation of parameters.

During this phase, information from published literature (health agencies or medical databases) is gathered with respect to the epidemiology of measles disease. Model parameters including the transmission rate, recovery rate, birth rate, death rate and vaccination rate are estimated from actual data. A key point, which is important, is that parameter estimation is an integral part of the mathematical model, and is directly related to the reliability and prediction capability of the mathematical model.

Step 3: Model Formulation:

After collecting the data, a compartmental mathematical model is developed. Population is comparted mentalized into Susceptible (S), Infected (I), Recovered (R). The movement of individuals between compartments is then represented by differential equations as a function of time. This will give the mathematical basis for studying the dynamics of the spread of measles.

The Caputo-type fractional derivative operator is then applied to the equations obtained in Step 3. Let us apply the Caputo-type fractional derivative operator to the equation obtained in step 3.

In this important step, the classical integer-order derivatives in the model are replaced with the Caputo fractional derivative operator. The fractional derivative captures the memory and hereditary effects in the system, where the current state of the disease is related not only with the present state but also to the disease history. A fractional order α ($0<\alpha\leq 1$) is used to model the memory effect and increase the realism and flexibility of the disease model.

Step 5: Model Analysis:

After the fractional model is formulated, mathematical analysis of the model is carried out to confirm the validity of the model. Checking positivity, checking boundedness of solutions, finding equilibrium points and local and global stability analysis are included. These analyses can help identify the stability of the disease-free and endemic states under various conditions.

Step 6: Numerical Solution:

Due to the difficulty in solving fractional differential equations, numerical methods are used. Approximate numerical solutions are obtained with methods like the Adams–Bashforth–Moulton method. Simulations are then carried out for various values of the fractional order α to observe changes in disease dynamics and prediction behavior under memory effects.

Step 7: Results and Discussion:

The phase consists of the analyses and interpretations of the numerical results of the simulation. The behavior of the fractional-order model is compared with that of the classical integer-order model through graphs and tables. The effects of changing the fractional order α on infection spread, recovery patterns and on the stability of the disease are commented on.

These are some definitions and lemma's which we use in our mathematical model. These definitions and lemmas provide the mathematical foundation for the proposed model. The Riemann–Liouville fractional integral is used in biomathematics and control systems.

Caputo-type fractional derivatives are used for modelling real-world problems in biomathematics in the biomathematics field. Caputo fractional derivative is also used for the approximation rate of the mathematical model.

Basic definitions [19][20][21] to define proposed mathematical model have been presented below:

Definition 1. Consider X **∈ I¹([0, ∞) R**) is said to be a function of Riemann-Liouville fractional integral with the order φ can be defined as the right hand integral exist.

$$I^{\varphi}{}_{t}X(t) = \frac{1}{\Gamma(\Phi)}\int_0^t X(\beta)\frac{1}{(t-\alpha)}d\alpha, \quad \varphi>0,$$

Definition 2. Let X be a function then the Caputo fractional order can be defines as

$$^{C}D_t^{\Phi}X(t) = \frac{1}{\Gamma(q-\theta)}\int_0^t (t-\gamma)^{q-1-\Phi}X^q(\gamma)\,d\gamma$$

The right side of the integral exist and q=Φ+1. If $\Phi \in (0,1)$, then the one is

$$^{C}D_t^{\Phi}X(t) = \frac{1}{\Gamma(1-\Phi)}\int_0^t X'(\Phi)\frac{1}{(t-\theta)^{\theta}}d\theta$$

Lemma 3. In fractional differential equations the given equation is:

$$I^{\Phi}[\,{}^{C}D_t^{\Phi}n(t) = n(t) + c_0 + c_1(t) + c_2(t^2) + \ldots\ldots c_{q-1}t^{q-1}$$

Definition 4. In Caputo formation the Laplace transform are given as

$$L[\,{}^{C}D_t^{\Phi}r(t)] = p^{\Phi}Q(s) - \sum_{i=0}^{n-1} p^{\Phi-k-1}\,q^k\,(0)$$
$$n-1 < \Phi < n \qquad n \in N$$

Data Analysis:

These are some parameters which we used in our mathematical model under caputo type fractional order of the measles disease. Because parameters is an important component of statistical analysis and these parameters refers to the characteristics which are used to define the entire populations.

Table 1. Parameters used in the model and its Description.

Parameter Notations	Parameters Description
Ψ	Birth/recruitment rate
ϒ	The proportion MDV vaccine
λ	The average death rate
ß	Death count as a result of MD
ε_{ψ}	Rate of contact between infected and uninfected people
α	Recovery time for an acutely infected person
ϰ	Recovering rate of a chronically infected person
θ	The rate at which an infected population transitions from an acute to a chronic state

After that the general series for the model system using the initial conditions under the caputo type fractional order has been generated [22][20][23][1]. Using a compartmental Susceptible-Exposed-Infectious-Recovered epidemic model with therapy, the mathematical measles disease model was created, and the mathematical analysis and numerical simulations that went along with it were clearly explained.

Model:

$$\begin{aligned}\frac{dL(t)}{dt} &= \Psi - \varepsilon_{\psi}L(t)M(t) - (\gamma + \lambda)L(t)\\ \frac{dM(t)}{dt} &= \varepsilon_{\psi} - L(t)M(t) - (\lambda + \alpha + \theta)L(t)\\ \frac{dU(t)}{dt} &= \theta M(t) - (\lambda + \beta + \varkappa)U(t)\end{aligned} \qquad \textbf{(1)}$$

$$\frac{dV(t)}{dt} = \alpha M(t) + \varkappa U(t) + \gamma L(t) - \lambda V(t).$$

The description of the parameters in the table 1. We will observe the model in fractional order in the sense of Caputo derivative as

$$\begin{aligned} {}^{c}D^{\varphi}{}_{t\ L(t)} &= \Psi - \varepsilon L(t)M(t) - (\gamma + \lambda)L(t) \\ {}^{c}D^{\varphi}{}_{t\ M(t)} &= \varepsilon L(t)M(t) - (\lambda + \alpha + \theta)M(t) \\ {}^{c}D^{\varphi}{}_{t\ U(t)} &= \theta M(t) - (\lambda + \beta + \varkappa)U(t) \\ {}^{c}D^{\varphi}{}_{t\ V(t)} &= \alpha M(t) + \varkappa U(t) + \gamma L(t) - \lambda V(t). \end{aligned} \qquad \textbf{(2)}$$

Initial Conditions:

$L(0) = H_1$

$M(0) = H_2$

$U(0) = H_3$

$V(0) = H_4$

Subject to the initial condition $L(0) = H_1$, $M(0) = H_2$, $U(0) = H_3$, $V(0) = H_4$. In classical calculus, Riemann–Liouville, In the 18th century, "Euler and Fourier" made significant contributions to the discovery of useful results [2][3][13][7]. Fractional calculus was only discovered by a handful of scholars during this time period (FC). Calculus resources handed down the generations do not adequately convey how modern calculus is used in many disciplines of mathematical modelling, such as memory processes and various mathematical models. It is a generic example with a particular instance of integer-order calculus, which has much more freedom in its derivative operators than the integer order of derivatives due to the locality condition in modern mathematics. A growing number of academics and researchers are focusing on the applications of differential and integral calculus of integer order (FO). For the first time, nonlocal and nonsingular kernels were used instead of single and local kernels in numerous articles in FC. The memory property and the system's genetics are the best features of this newly developed kernel. Some of the arbitrary order differential operators differ in numerous ways. No ordinary kernel exists for definite integration, for example, hence it has been possible to find definitions that include both a singularity and non-singularity kernel. When discussing the prior definitions of arbitrary order of derivatives, many more solutions were available in the literature. Many nonlinear models could be solved analytically or semi-analytically using decomposition techniques and homotopy perturbation methods (HPM). To study mathematical models for approximate solutions in general, Runge–Kutta procedures (RK4 and RzK2) were applied. In our work, we employ approximate or semi-analytical solutions. Series solutions can be found by splitting the desired quantity into small integers using fractional Laplace transforms with Adomian polynomials in nonlinear terms. On the other hand, homotopy perturbation approaches were incorporated into numerical simulations, and their outcomes were compared. Decomposition methods are combined to construct Laplace's Adomain decomposition method. The Laplace transform is used to turn differential equations into algebraic equations and then use Adomain polynomials to decompose the nonlinear terms. For both deterministic and stochastic differential equations, this numerical approach is very efficient. Furthermore, it may be applied to a system of ordinary and partial differential equations of both classical and fractional order. This approach does not call for any alterations or alterations of any kind. Furthermore, unlike RK4, it does not necessitate a predetermined step size. Note that the LADM method is more powerful than the conventional ADM method.

General solution for the construction of Model (2) through LADM:

The model system's beginning conditions are used in this part to generate the general series under the caputo type fractional order by applying a Laplace transform of the two sides of the model with the aid of LADM. Nonlinear ordinary and partial differential equations can be precisely solved using an analytical technique called the Laplace Adomian decomposition

method (LADM).This technique provide the accurate solution of nonlinear ordinary and partial differential equation.

$$L[{}^{c}D^{\varphi}_{t}(L(t)] = L[\Psi - \varepsilon_{\downarrow}LM - (\gamma + \lambda)L]$$
$$L[{}^{c}D^{\varphi}_{t}(M(t)] = L[\varepsilon_{\downarrow}LM - (\lambda + \alpha + \theta)M] \quad \ldots\ldots\ldots(3)$$
$$L[{}^{c}D^{\varphi}_{t}(U(t)] = L[\theta M - (\lambda + \beta + \varkappa)U]$$
$$L[{}^{c}D^{\varphi}_{t}(V(t)] = L[\alpha M + \varkappa U + \gamma L - \lambda V]$$

Using the initial condition in **equation (3)** and solve these equations.

Now we take a laplace transform of eq (3(i)) using the initial conditions and then get the solution.

$$\mathbf{L[{}^{c}D^{\varphi}_{t}(L(t)] = L[\Psi - \varepsilon_{\downarrow}LM - (\gamma + \lambda)L] \ (A)}$$

Solving L.H.S of equation (A)

$$L[{}^{c}D^{\varphi}_{t}(L(t)] = s^{\Phi}\,(\overline{L}(r,s) - \overline{L}(r,0))$$

Initial condition.

$L(0) = H_1$

$\overline{L}(0) = \frac{H1}{s}$

$$L[D_t^{\Phi}(L(t)] = s^{\Phi}\,(\overline{L}(r,s) - \frac{H_1}{s})$$

Put value in equation ----- (A)

$$s^{\Phi}(\overline{L}(r,s) - \frac{H_1}{s}) = L[\Psi - \varepsilon_{\downarrow}LM - (\gamma + \lambda)L]$$
$$\overline{L}(r,s) - \frac{H_1}{s} = \frac{1}{s^{\Phi}} L[\Psi - \varepsilon_{\downarrow}LM - (\gamma + \lambda)L]$$
$$\overline{L}(r,s) = \frac{H_1}{s} + \frac{1}{s^{\Phi}} L[\Psi - \varepsilon_{\downarrow}LM - (\gamma + \lambda)L]$$
$$L[L(t)] = \frac{H_1}{s} + \frac{1}{s^{\Phi}} L[\Psi - \varepsilon_{\downarrow}LM - (\gamma + \lambda)L] \quad \text{---------- (i)}$$

Now, Now we take a laplace transform of eq (3(ii)) using yhe initial conditions and then get the solution.

$$\mathbf{L[{}^{c}D^{\varphi}_{t}(M(t)] = L[\varepsilon_{\downarrow}LM - (\lambda + \alpha + \theta)M] \ (B)}$$

Solving L.H.S of equation (B)

$$L[{}^{c}D^{\varphi}_{t}(M(t)] = s^{\Phi}\,(\overline{M}(r,s) - \overline{M}(r,0))$$

Initial condition

$$M(0) = H_2$$
$$\overline{M}(0) = \frac{H2}{s}$$
$$L[{}^{c}D^{\varphi}_{t}(M(t)] = s^{\Phi}\,(\overline{M}(r,s) - \frac{H_2}{s})$$

Put value in equation ------ (B) and then get the solution.

$$s^{\Phi}\,(\overline{M}(r,s) - \frac{H_2}{s}) = L[\varepsilon_{\downarrow}LM - (\lambda + \alpha + \theta)M]$$
$$\overline{M}(r,s) - \frac{H_2}{s} = \frac{1}{s^{\Phi}} L[\varepsilon_{\downarrow}LM - (\lambda + \alpha + \theta)M]$$
$$\overline{M}(r,s) = \frac{H_2}{s} + \frac{1}{s^{\Phi}} L[\varepsilon_{\downarrow}LM - (\lambda + \alpha + \theta)M]$$
$$L[M(t)] = \frac{H_2}{s} + \frac{1}{s^{\Phi}} L[\varepsilon_{\downarrow}LM - (\lambda + \alpha + \theta)M] \quad \text{-------- (ii)}$$

Now, we take a laplace transform of eq (3(iii)) using the initial conditions and then get the solution.

$$\mathbf{L[{}^{c}D^{\varphi}_{t}(U(t)] = L[\theta M - (\lambda + \beta + \varkappa)U] \quad \text{--------(C)}}$$

Solving L.H.S of equation (C)

$$L[{}^{c}D^{\varphi}_{t}(U(t)] = s^{\Phi}\,(\overline{U}(r,s) - \overline{G}(r,0))$$

Initial condition

$$U(0) = H_3$$

$$\bar{U}(0) = \frac{H_3}{s}$$

$$L[{}^{c}D^{\varphi}_{t}(U(t)] = s^{\Phi}\left(\bar{U}(r,s) - \frac{H_3}{s}\right)$$

Put value in equation ------ (C)

$$s^{\Phi}\left(\bar{U}(r,s) - \frac{H_3}{s}\right) = L[\theta M - (\lambda+\beta+\varkappa)U]$$

$$\bar{U}(r,s) - \frac{H_3}{s} = \frac{1}{s^{\Phi}} L[\theta M - (\lambda+\beta+\varkappa)U]$$

$$\bar{U}(r,s) = \frac{H_3}{s} + \frac{1}{s^{\Phi}} L[\theta M - (\lambda+\beta+\varkappa)U]$$

$$L[U(t)] = \frac{H_3}{s} + \frac{1}{s^{\Phi}} L[\theta M - (\lambda+\beta+\varkappa)U] \quad \text{----------(iii)}$$

Now, we take a laplace transform of eq (3(iv)) using yhe initial conditions and then get the solution.

$$\mathbf{L[{}^{c}D^{\varphi}_{t}(V(t)] = L[\alpha M + \varkappa U + \dot{\gamma} L - \lambda V]} \quad \mathbf{----------(D)}$$

Solving L.H.S of equation (D)

$$L[{}^{c}D^{\varphi}_{t}(V(t)] = s^{\Phi}\left(\bar{V}(r,s) - \bar{V}(r,0)\right)$$

Initial condition

$$V(0) = H_4$$

$$\bar{V}(0) = \frac{H_4}{s}$$

$$L[{}^{c}D^{\varphi}_{t}(V(t)] = s^{\Phi}\left(\bar{V}(r,s) - \frac{H_4}{s}\right)$$

Put value in equation ------ (D)

$$s^{\Phi}\left(\bar{V}(r,s) - \frac{H_4}{s}\right) = L[\alpha M + \varkappa U + \dot{\gamma} L - \lambda V]$$

$$\bar{V}(r,s) - \frac{H_4}{s} = \frac{1}{s^{\Phi}} L[\alpha M + \varkappa U + \dot{\gamma} L - \lambda V]$$

$$\bar{Q}(r,s) = \frac{H_4}{s} + \frac{1}{s^{\Phi}} L[\alpha M + \varkappa U + \dot{\gamma} L - \lambda V]$$

$$L[V(t)] = \frac{H_4}{s} + \frac{1}{s^{\Phi}} L[\alpha M + \varkappa U + \dot{\gamma} L - \lambda V] \quad \text{---------- (iv)}$$

Now, we write the equation (i), (ii), (iii) & (iv) using initial conditions with the help of laplace transform.

$$L[L(t)] = \frac{H_1}{s} + \frac{1}{s^{\Phi}} L[\Psi - \varepsilon_{\dot{g}} LM - (\dot{\gamma} + \lambda)L]$$

$$L[M(t)] = \frac{H_2}{s} + \frac{1}{s^{\Phi}} L[\varepsilon_{\dot{g}} LM - (\lambda + \alpha + \theta)M] \quad \text{..... (4)}$$

$$L[U(t)] = \frac{H_3}{s} + \frac{1}{s^{\Phi}} L[\theta M - (\lambda+\beta+\varkappa)U]$$

$$L[V(t)] = \frac{H_4}{s} + \frac{1}{s^{\Phi}} L[\alpha M + \varkappa U + \dot{\gamma} L - \lambda V]$$

After getting solution of Caputo derivative using initial conditions under the the laplace transform we assume the infinite series of **L , M , U & V** as

$$L(t) = \sum_{q=0}^{\infty} L_q(t) \quad , \quad M(t) = \sum_{q=0}^{\infty} M_q(t)$$

$$U(t) = \sum_{q=0}^{\infty} U_q(t) \quad , \quad V(t) = \sum_{q=0}^{\infty} V_q(t) \quad \text{.....(5)}$$

Here the non-linear term of L(t) M(t) is given as

$$L(t)M(t) = \sum_{q=0}^{\infty} Z_q(t)$$

$$Z_q(t) = \frac{1}{q!} \frac{s^m}{s\,\theta^m} \left[\sum_{l=0}^{\infty} \theta^l L_l(t) \sum_{l=0}^{\infty} \theta^l M_l(t)\right]_{\theta=0} \quad \text{.....(6)}$$

These are Laplace transforms of a constant values which we use in our proposed model system using the initial conditions.

$$L[L_0(t)] = \frac{H_1}{s} \quad , \quad L[M_0(t)] = \frac{H_2}{s}$$

$$L[\,U_0(t)\,] = \frac{H_3}{s} \quad , \quad L[\,V_0(t)\,] = \frac{H_4}{s}$$

Substituting equation (5) and (6) in equation (4) and comparing term of both sides and then we get the solution in the form of Laplace transform.

$$L[\,L_1\,(t)\,] = \frac{1}{s^{\Phi}} L[\Psi - \varepsilon_{\circ} L_0\, M_0 - (\nu+\lambda)\, L_0]$$

$$L[M_1\,(t)\,] = \frac{1}{s^{\Phi}} L[\varepsilon_{\circ} L_0\, M_0 - (\lambda+\alpha+\theta) M_0]$$

$$L[\,U_1\,(t)\,] = \frac{1}{s^{\Phi}} L[\theta M_0 - (\lambda+\kappa+\beta)\, U_0]$$

$$L[V_1\,(t)] = \frac{1}{s^{\Phi}} L[\alpha M_0 + \kappa\, U_0 + \nu\, L_0 - \lambda V_0]$$

$$L[\,L_{q+1}\,(t)\,] = \frac{1}{s^{\Phi}} L[\Psi - \varepsilon_{\circ} L_q\, M_q - (\nu+\lambda)\, L_q]$$

$$L[M_{q+1}\,(t)\,] = \frac{1}{s^{\Phi}} L[\varepsilon_{\circ} L_q\, M_q - (\lambda+\alpha+\theta) M_q]$$

$$L[\,U_{q+1}\,(t)\,] = \frac{1}{s^{\Phi}} L[\theta M_q - (\lambda+\kappa+\beta)\, U_q] \qquad \text{..... (7)}$$

$$L[V_{q+1}\,(t)] = \frac{1}{s^{\Phi}} L[\alpha M_q + \kappa\, U_q + \nu\, L_q - \lambda V_q]$$

Use the laplace and laplace inverse to **equation (7)** and solve these equations one by one.

$$[\,L_0(t)\,] = L^{-1}\left[\frac{H_1}{s}\right] \quad , \quad L^{-1}\left[\frac{1}{s}\right] = s$$

$$[\,L_0(t)\,] = H_1 \qquad \text{.....(i)}$$

$$[\,M_0(t)\,] = L^{-1}\left[\frac{H_2}{s}\right]$$

$$\text{So , } [\,M_0(t)\,] = H_2 \qquad \text{.....(ii)}$$

$$[\,U_0(t)\,] = L^{-1}\left[\frac{H_3}{s}\right]$$

$$\text{So } [\,U_0(t)\,] = H_3 \qquad \text{.....(iii)}$$

$$[\,V_0(t)\,] = L^{-1}\left[\frac{H_4}{s}\right]$$

$$[\,V_0(t)\,] = H_4 \qquad \text{.....(iv)}$$

We solve the equations with the help of Eq (7)

$$L[\,L_1\,(t)\,] = \frac{1}{s^{\Phi}} L[\Psi - \varepsilon_{\circ} L(t) M(t) - (\nu+\lambda)\, L_0]$$

Solving the laplace transformation on both sides and get a new equation.

$$L(1) = 1/s$$

$$L[\,L_0(t)\,] = \frac{H_1}{s} \quad , \quad L[\,M_0(t)\,] = \frac{H_2}{s}$$

$$\frac{[\,L_1\,(t)\,]}{s} = \frac{1}{s^{\Phi}}\left[\frac{\Psi}{s} - \frac{\varepsilon_{\circ}}{s}\left[\frac{H_1}{s}\right]\left[\frac{H_2}{s}\right] - \frac{(\nu+\lambda)}{s}\left[\frac{H_1}{s}\right]\right]$$

Using laplace inverse on both sides and solve it one by one equation.

$$L^{-1}\left[\frac{1}{s}\right] = s \quad \text{and} \quad \frac{1}{s^{\Phi}} = \frac{t^{\Phi-1}}{\Gamma(\Phi)}$$

$$[\,L_1\,(t)\,] = [\Psi - \varepsilon_{\circ} H_1 H_2 - (\nu+\lambda) H_2] * \frac{t^{\Phi-1}}{\Gamma(\Phi)}$$

$$[\,L_1\,(t)\,] = [\Psi - \varepsilon_{\circ} H_1 H_2 - (\nu+\lambda) H_2] \frac{t^{\Phi-1+1}}{\Gamma(\Phi+1)}$$

$$[\,L_1\,(t)\,] = [\Psi - \varepsilon_{\circ} H_1 H_2 - (\nu+\lambda) H_2] * \frac{t^{\Phi}}{\Gamma(\Phi+1)} \qquad \text{.....(v)}$$

We solve the equations with the help of Eq (7)

$$L[M_1(t)] = \frac{1}{s^{\Phi}} L[\varepsilon_{\circ} L_0 M_0 (\lambda+\alpha+\theta) M_0]$$

Solving the laplace transformation on both sides and get a new equation.

$$L(1) = 1/s$$

$$L[\,L_0(t)\,] = \frac{H_1}{s} \quad , \quad L[\,M_0(t)\,] = \frac{H_2}{s}$$

$$\frac{[M_1(t)]}{s} = \frac{1}{s^{\Phi}}\left[\frac{\varepsilon_0}{s}\left[\frac{H_1}{s}\right]\left[\frac{H_2}{s}\right] - \frac{(\lambda+\alpha+\theta)}{s}\left[\frac{H_2}{s}\right]\right]$$

Using laplace inverse on both sides and solve it one by one equation.

$$\text{As; } L^{-1}\left[\frac{1}{s}\right] = s \quad \text{and} \quad \frac{1}{s^{\Phi}} = \frac{t^{\Phi-1}}{\Gamma(\Phi)}$$

$$[M_1(t)] = [\varepsilon_0 E_1 E_2 - (\lambda+\alpha+\theta)E_2]^* \frac{t^{\Phi-1}}{\Gamma(\Phi)}$$

$$[M_1(t)] = [\varepsilon_0 E_1 E_2 - (\lambda+\alpha+\theta)E_2]^* \frac{t^{\Phi-1+1}}{\Gamma(\Phi+1)}$$

$$[M_1(t)] = [\varepsilon_0 E_1 E_2 - (\lambda+\alpha+\theta)E_2]^* \frac{t^{\Phi}}{\Gamma(\Phi+1)} \qquad \textbf{.....(vi)}$$

We solve the equations with the help of Eq (7)

$$L[U_1(t)] = \frac{1}{s^{\Phi}} L[\theta M_0 - (\lambda+\beta+\varkappa)U_0]$$

Solving the laplace transformation on both sides and get a new equation.

$$L(1) = 1/s$$

$$L[M_0(t)] = \frac{H_2}{s}, \qquad L[U_0(t)] = \frac{H_3}{s}$$

$$\frac{[U_1(t)]}{s} = \frac{1}{s^{\Phi}}\left[\frac{\theta}{s}\left[\frac{H_2}{s}\right] - \frac{(\lambda+\beta+\kappa)}{s}\left[\frac{H_3}{s}\right]\right]$$

Using laplace inverse on both sides and solve it one by one equation.

$$L^{-1}\left[\frac{1}{s}\right] = s \quad \text{and} \quad \frac{1}{s^{\Phi}} = \frac{t^{\Phi-1}}{\Gamma(\Phi)}$$

$$[U_1(t)] = [\theta H_2 - (\lambda+\beta+\kappa)H_3]^* \frac{t^{\Phi-1}}{\Gamma(\Phi)}$$

$$[U_1(t)] = [\theta H_2 - (\lambda+\beta+\kappa)H_3]^* \frac{t^{\theta-1+1}}{\Gamma(\theta+1)}$$

$$[U_1(t)] = [\theta H_2 - (\lambda+\beta+\kappa)H_3]^* \frac{t^{\Phi}}{\Gamma(\Phi+1)} \qquad \text{....(vii)}$$

Solving the laplace transformation on both sides and get a new equation.

$$L[V_1(t)] = \frac{1}{s^{\Phi}} L[\alpha M_0 + \varkappa U_0 + \gamma L_0 - \lambda V_0]$$

Solving the Laplace on both size

$$L(1) = 1/s$$

$$L[L_0(t)] = \frac{H_1}{s} L[M_0(t)] = \frac{H_2}{s}$$

$$L[U_0(t)] = \frac{H_3}{s}, \qquad L[V_0(t)] = \frac{H_4}{s}$$

$$\frac{[V_1(t)]}{s} = \frac{1}{s^{\Phi}}\left[\frac{\alpha}{s}\left[\frac{H_2}{s}\right] + \frac{\kappa}{s}\left[\frac{H_3}{s}\right] + \frac{\gamma}{s}\left[\frac{H_1}{s}\right] - \frac{\lambda}{s}\left[\frac{H_4}{s}\right]\right]$$

Using Laplace inverse on both sides and solve it.

$$\text{As; } L^{-1}\left[\frac{1}{s}\right] = s \quad \text{and} \quad \frac{1}{s^{\Phi}} = \frac{t^{\Phi-1}}{\Gamma(\Phi)}$$

Using laplace inverse on both sides and solve it one by one equation.

$$\text{As; } L^{-1}\left[\frac{1}{s}\right] = s \quad \text{and} \quad \frac{1}{s^{\Phi}} = \frac{t^{\Phi-1}}{\Gamma(\Phi)}$$

$$[V_1(t)] = [\alpha H_2 + \kappa H_3 + \gamma H_1 - \lambda H_4]^* \frac{t^{\Phi-1}}{\Gamma(\Phi)}$$

$$[V_1(t)] = [\alpha H_2 + \kappa H_3 + \gamma H_1 - \lambda H_4]^* \frac{t^{\Phi-1+1}}{\Gamma(\Phi+1)}$$

$$[V_1(t)] = [\alpha H_2 + \kappa H_3 + \gamma H_1 - \lambda H_4]^* \frac{t^{\Phi}}{\Gamma(\Phi+1)}$$

Solve this equation from (7) to find the next term.

$$\mathbf{L[L_{q+1}(t)] = \frac{1}{s^{\Phi}} L[\Psi - \varepsilon_0 L_q M_q - (\gamma+\lambda) L_q]}$$

Put q=1;

$$L[\,L_2\,(t)\,] = \frac{1}{s^{\Phi}}\,L[\,\Psi - \varepsilon_{\circ} L_1\,M_1\, - (\dot{\gamma}+\lambda)\,L_1]$$

$$L[\,L_2\,(t)\,] = \frac{1}{s^{\Phi}}\,[L\,(\Psi) - L\,(\varepsilon_{\circ} L_1\,M_1)\, - L\,(\dot{\gamma}+\lambda)\,L(L_1)]$$

Now, put the values of $L(L_1)$ and $L(M_1)$

$$L[\,L_2\,(t)\,] = \frac{1}{s^{\Phi}}\,[L\,(\Psi) - L\,(\varepsilon_{\circ}) * \frac{1}{s^{\Phi}}\,L[\Psi - \varepsilon_{\circ} L_0\,M_0 - (\dot{\gamma}+\lambda)\,L_0]*\frac{1}{s^{\Phi}}\,L[\varepsilon_{\circ} L_0\,M_0 - (\lambda+\alpha+\theta)M_0] -$$

$$L\,(\dot{\gamma}+\lambda)\,\frac{1}{s^{\Phi}}\,L[\Psi-\varepsilon_{\circ} L_0\,M_0\, - (\dot{\gamma}+\lambda)\,L_0]$$

Solving Laplace on both sides

$$\frac{[\,L_2\,(t)\,]}{s} = \frac{1}{s^{\Phi}}\frac{(\Psi)}{s} - [\frac{(\varepsilon)}{s} * [\frac{(\Psi)}{s} - \frac{(\varepsilon)}{s}\frac{(H_1)}{s}\frac{(H_2)}{s} - \frac{(\dot{\gamma}+\lambda)}{s}\frac{(H_1)}{s}]*[\frac{(\varepsilon)}{s}\frac{(H_1)}{s}\frac{(H_2)}{s} - \frac{(\lambda+\alpha+\theta)}{s}\frac{(H_2)}{s}\,] - \frac{(\dot{\gamma}+\lambda)}{s} * \frac{(\Psi)}{s} -$$

$$\frac{(\varepsilon)}{s}\frac{(H_1)}{s}\frac{(H_2)}{s} - \frac{(\dot{\gamma}+\lambda)}{s}\frac{(H_1)}{s}] * \frac{1}{s^{2\Phi}}$$

Apply Laplace inverse on the both sides

$$L^{-1}\left[\frac{1}{s}\right] = 1 \quad \text{and} \quad \frac{1}{s^{2\Phi}} = \frac{t^{2\Phi}}{\Gamma\,(2\Phi+1)}$$

$$[\,L_2\,(t)\,] = \Psi\frac{t^{\Phi}}{\Gamma\,(\Phi+1)} - [(\Psi - \varepsilon_{\circ} H_1 H_2\, - (\dot{\gamma}+\lambda)H_1) * (\,\varepsilon_{\circ} H_1 H_2\, - (\lambda+\alpha+\theta)H_2) - (\dot{\gamma}+\lambda) *$$

$$(\Psi - \varepsilon_{\circ} H_1 H_2\, - (\dot{\gamma}+\lambda)H_1)\,] * \frac{t^{2\Phi}}{\Gamma\,(2\Phi+1)}$$

$$[\,L_2\,(t)\,] = \Psi\frac{t^{\Phi}}{\Gamma\,(\Phi+1)} - [\varepsilon_{\circ}(W_{11}X_{11}) - (\dot{\gamma}+\lambda)\,W_{11}]* \frac{t^{2\Phi}}{\Gamma\,(2\Phi+1)}$$

$$\mathbf{L[M_{q+1}\,(t)\,] = \frac{1}{s^{\Phi}}\,L[\varepsilon_{\circ} L_q\,M_q - (\lambda+\alpha+\theta)M_q]}$$

Put q= 1, …

$$L[M_2(t)] = \frac{1}{s^{\Phi}}\,L[\varepsilon_{\circ} L_1\,M_1 - (\lambda+\alpha+\theta)M_1]$$

$$L[M_2(t)] = \frac{1}{s^{\Phi}}[\,L(\varepsilon_{\circ} L_1\,M_1) - L\,(\lambda+\alpha+\theta)L(M_1)]$$

By putting values of $L(L_1)$ and $L(M_1)$……

$$L[M_2(t)] = \frac{1}{s^{\Phi}}[\,L(\varepsilon_{\circ}\frac{1}{s^{\Phi}}L((\Psi - \varepsilon_{\circ} L_0 M_0 - (\dot{\gamma}+\lambda)L_0)\,\frac{1}{s^{\Phi}}\,((\varepsilon_{\circ} - L_0 M_0 - (\lambda+\alpha+\theta)M_0) - L(\lambda+\alpha+\theta)\,\frac{1}{s^{\Phi}}$$

$$L(\varepsilon_{\circ} L_0 M_0 - (\lambda+\alpha+\theta)M_0)$$

Solving the Laplace Transform on both sides

$$\frac{M_2(t)}{s} = = \frac{1}{s^{2\Phi}}\,[\,\frac{\varepsilon}{s}\,[\frac{\Psi}{s}\frac{H_1}{s}\frac{H_2}{s} - \frac{(\dot{\gamma}+\lambda)}{s}\frac{H_1}{s}]\,[\frac{\Psi}{s}\frac{H_1}{s}\frac{H_2}{s} - \frac{(\lambda+\alpha+\theta)}{s}\,] - \frac{(\lambda+\alpha+\theta)}{s}\,[\frac{\Psi}{s}\frac{H_1}{s}\frac{H_2}{s} - \frac{(\lambda+\alpha+\theta)}{s}\frac{H_2}{s}]$$

Now Apply Laplace inverse on both ides.

$$L^{-1}\,[\frac{1}{s}] = 1\;\; \frac{1}{s^{2\Phi}} = \frac{t^{2\Phi}}{\Gamma\,(2\Phi+1)}$$

$$M_2(t) = [\varepsilon_{\circ}[\Psi - \varepsilon_{\circ} H_1 H_2 - (\dot{\gamma}+\lambda)H_1][\,\varepsilon_{\circ} H_1 H_2 - (\,(\lambda+\alpha+\theta)H_2] - [(\lambda+\alpha+\theta)[\varepsilon_{\circ} H_1 H_2 -$$

$$(\lambda+\alpha+\theta)H_2]\frac{t^{2\Phi}}{\Gamma\,(2\Phi+1)}$$

$$M_2(t) = [\varepsilon_{\circ}(W_{11}X_{11}) - ((\lambda+\alpha+\theta)X_{11}]\,\frac{t^{2\Phi}}{\Gamma\,(2\Phi+1)}$$

$$\mathbf{L[\,U_{q+1}\,(t)\,] = \frac{1}{s^{\Phi}}L[\theta M_q - (\lambda+\kappa+\beta)\,U_q]}$$

Put q= 1, …

$$L[\,U_2\,(t)\,] = \frac{1}{s^{\Phi}}L[\theta M_1 - (\lambda+\kappa+\beta)\,U_1]$$

$$L[\,U_2\,(t)\,] = \frac{1}{s^{\Phi}}[L(\theta)L(M_1) - L\,(\lambda+\kappa+\beta)L\,U_1]$$

By putting values of $L(M_1)$ an $L(U_1)$

$$L[\,U_2\,(t)\,] = \frac{1}{s^{\Phi}}[L(\theta)\frac{1}{s^{\Phi}}L[\varepsilon_{\circ} L_0 M_0 - (\lambda+\alpha+\theta)M_0] - L(\lambda+\kappa+\beta)\,\frac{1}{s^{\Phi}}\,L[\theta\,M_0 - (\lambda+\beta+\varkappa)U_0]$$

Solving the Laplace transform on both sides .

$$\frac{U_2(t)}{s} = = \frac{1}{s^{2\Phi}}\left[\frac{\theta}{s}\left[\frac{\Psi}{s}\frac{H_1}{s}\frac{H_2}{s} - \frac{(\lambda+\alpha+\theta)}{s}\frac{H_2}{s}\right] - \frac{(\lambda+\alpha+\theta)}{s}\right] - \frac{(\lambda+\kappa+\beta)}{s}\left[\frac{\Psi}{s}\frac{H_2}{s} - \frac{(\lambda+\kappa+\beta)}{s}\frac{H_3}{s}\right]$$

Now Apply Laplace inverse on both sides.

$$L^{-1}\left[\frac{1}{s}\right] = 1 \quad \frac{1}{s^{2\Phi}} = \frac{t^{2\Phi}}{\Gamma(2\Phi+1)}$$

$$L[U_2(t)] = [\theta(\Psi H_1H_2 - (\lambda+\alpha+\theta)H_2] - ((\lambda+\kappa+\beta)[\theta H_2 - (\lambda+\alpha+\theta)H_3]\frac{t^{2\Phi}}{\Gamma(2\Phi+1)}$$

$$U_2(t) = [\theta X_{11} - ((\lambda+\kappa+\beta)Y_{11}$$

Similarly the Laplace transform of V to find the next term.

$$\mathbf{L[V_{q+1}(t)] = \frac{1}{s^{\Phi}}L[\alpha M_q + \kappa U_q + \gamma L_q - \lambda V_q]}$$

By solving the Laplace transform of V we obtain the solution.

$$V_2(t) = [\alpha X_{11} + \varkappa Y_{11} + \gamma W_{11} + \lambda Z_{11}]\frac{t^{2\Phi}}{\Gamma(2\Phi+1)}$$

$L_2(t)$, $M_2(t)$, $U_2(t)$, $V_2(t)$ are the equation (8)

The equation (7) show the infinite series of a Laplace transform and also we use the LADM technique to obtain the series solution of a proposed model system of the measles disease. By using LADM technique we obtain a better solution of reducing the effectivity rate of the measles disease in the human being.

General solution for the construction of Model (2) through HPM:

Using the HPM technique, we'll figure out the series solution for model (2) and then apply initial conditions under Caputo type fractional order to derive the general series for the model system.

According to [12], [13] as follows.

$$(1-p)\left[{}^{c}D^{\varphi}_{t}(L(t)) - {}^{c}D^{\varphi}_{t}(L_0(t))\right] + p\left[{}^{c}D^{\varphi}_{t}(L(t)) - \Psi - \varepsilon_1 LM - (\gamma+\lambda)L\right] = 0, \text{ (i)}$$

$$(1-p)\left[{}^{c}D^{\varphi}_{t}(M(t)) - {}^{c}D^{\varphi}_{t}(X_0(t))\right] + p\left[{}^{c}D^{\varphi}_{t}(M(t)) - \varepsilon_1 LM + (\lambda+\alpha+\theta)M\right] = 0, \text{ (ii)}$$

$$(1-p)\left[{}^{c}D^{\varphi}_{t}(U(t)) - {}^{c}D^{\varphi}_{t}(Y_0(t))\right] + p\left[{}^{c}D^{\varphi}_{t}(U(t)) - \theta M + (\lambda+\beta+\varkappa)U\right] = 0, \text{ (iii)}$$

$$(1-p)\left[{}^{c}D^{\varphi}_{t}(V(t)) - {}^{c}D^{\varphi}_{t}(V_0(t))\right] + p\left[{}^{c}D^{\varphi}_{t}(V(t)) - \alpha M - \varkappa U - \gamma L + \lambda V\right] = 0, \quad \text{(iv)}$$

The above equations is Eq(9(i,ii,iii,iv)).

By using p=0 in equation (9(i,ii,iii,iv)). a system of FDEs can be obtained as.

$$(1-0)\left[{}^{c}D^{\varphi}_{t}(L(t)) - {}^{c}D^{\varphi}_{t}(L_0(t))\right] + 0\left[{}^{c}D^{\varphi}_{t}(L(t)) - \Psi - \varepsilon_1 LM - (\gamma+\lambda)L\right] = 0,$$

$$\left[{}^{c}D^{\varphi}_{t}(L(t)) - {}^{c}D^{\varphi}_{t}(L_0(t))\right] = 0.$$

$$(1-0)\left[{}^{c}D^{\varphi}_{t}(M(t)) - {}^{c}D^{\varphi}_{t}(X_0(t))\right] + 0\left[{}^{c}D^{\varphi}_{t}(M(t)) - \varepsilon_1 LM + (\lambda+\alpha+\theta)M\right] = 0$$

$$\left[{}^{c}D^{\varphi}_{t}(M(t)) - {}^{c}D^{\varphi}_{t}(X_0(t))\right] = 0.$$

$$(1-0)\left[{}^{c}D^{\varphi}_{t}(U(t)) - {}^{c}D^{\varphi}_{t}(Y_0(t))\right] + 0\left[{}^{c}D^{\varphi}_{t}(U(t)) - \theta M + (\lambda+\beta+\varkappa)U\right] = 0,$$

$$\left[{}^{c}D^{\varphi}_{t}(U(t)) - {}^{c}D^{\varphi}_{t}(Y_0(t))\right] = 0.$$

$$(1-0)\left[{}^{c}D^{\varphi}_{t}(V(t)) - {}^{c}D^{\varphi}_{t}(V_0(t))\right] + 0\left[{}^{c}D^{\varphi}_{t}(V(t)) - \alpha M - \varkappa U - \gamma L + \lambda V\right] = 0,$$

By putting the value of p=0; in the equation 9. We obtain the given solution are as follows.

$$\left[{}^{c}D^{\varphi}_{t}(V(t)) - {}^{c}D^{\varphi}_{t}(V_0(t))\right] = 0.$$

$$\left[{}^{c}D^{\varphi}_{t}(L(t)) - {}^{c}D^{\varphi}_{t}(L_0(t))\right] = 0.$$

$$\left[{}^{c}D^{\varphi}_{t}(M(t)) - {}^{c}D^{\varphi}_{t}(X_0(t))\right] = 0.(10)$$

$$\left[{}^{c}D^{\varphi}_{t}(U(t)) - {}^{c}D^{\varphi}_{t}(Y_0(t))\right] = 0.$$

$$\left[{}^{c}D^{\varphi}_{t}(V(t)) - {}^{c}D^{\varphi}_{t}(V_0(t))\right] = 0.$$

The solution to equation 10 are given above. By using p=1 in equation (9(i,iii,iii,iv)) a system of FDEs can be obtained as.

$$(1-1)\left[{}^{c}D^{\varphi}_{t}(M(t)) - {}^{c}D^{\varphi}_{t}(X_0(t))\right] + 1\left[{}^{c}D^{\varphi}_{t}(M(t)) - \varepsilon_1 LM + (\lambda+\alpha+\theta)M\right] = 0$$

$$\left[{}^{c}D^{\varphi}_{t}(M(t)) - \varepsilon_1 LM + (\lambda+\alpha+\theta)M\right] = 0$$

$$(1-1)\left[{}^{c}D^{\varphi}_{t}(M(t)) - {}^{c}D^{\varphi}_{t}(X_0(t))\right] + 1\left[{}^{c}D^{\varphi}_{t}(M(t)) - \varepsilon_1 LM + (\lambda+\alpha+\theta)M\right] = 0$$

$$\left[{}^{c}D^{\varphi}_{t}(M(t)) - \varepsilon_1 LM + (\lambda+\alpha+\theta)M\right] = 0$$

$$(1-1)\ [{}^{c}D^{\varphi}_{t}(U(t)) - {}^{c}D^{\varphi}_{t}(Y_0(t))] + 1[{}^{c}D^{\varphi}_{t}(U(t)) - \theta M + (\lambda+\beta+\varkappa)U] = 0,$$

$$[{}^{c}D^{\varphi}_{t}(U(t)) - \theta M + (\lambda+\beta+\varkappa)U] = 0,$$

$$(1-1)\ [{}^{c}D^{\varphi}_{t}(V(t)) - {}^{c}D^{\varphi}_{t}(V_0(t))] + 1[{}^{c}D^{\varphi}_{t}(V(t)) - \alpha M - \varkappa U - \gamma L + \lambda V]] = 0,$$

$$[{}^{c}D^{\varphi}_{t}(V(t)) - \alpha M - \varkappa U - \gamma L + \lambda V] = 0,$$

After getting solution of Caputo derivative using initial conditions under the the laplace transform we assume the infinite series of **L , M , U & V** as.

$$L(t) = \sum_{q=0}^{\infty} p^{q} L_q(t) \quad , \quad M(t) = \sum_{q=0}^{\infty} p^{q} M_q(t)$$

$$U(t) = \sum_{q=0}^{\infty} p^{q} U_q(t) \quad , \quad V(t) = \sum_{q=0}^{\infty} p^{q} V_q(t) \qquad \textbf{(11)}$$

Hence by using p=1 in equation (11) we obtained the original system. Putting Eq (11) into Eq (9) and we obtained the solution. Comparing the coefficient of 'p' and also the same power of 'p^{o}', 'p^{1}', 'p^{2}'.

Now comparing the coefficient of 'p^{o}'and the same power. We obtain the given values of 'p' raised to the power '0' and the same power.

$$\mathbf{p^{o}} = \{ L_o(t) = H_1 \quad , \quad M_o(t) = H_2 \quad , \quad U_o(t) = H_3 \quad , \quad V_o(t) = H_4.$$

These are the coefficient of 'p' raised to the power '0'.

Now comparing the coefficient of 'p^{1}'and the same power. We obtain the given values of 'p' raised to the power '1'.

$$L_1 = [\Psi - \varepsilon_{\downarrow} L_o M_o - (\gamma + \lambda) L_o] \frac{t\Phi}{\Gamma(\Phi+1)}$$

$$M_1 = [\varepsilon_{\downarrow} L_o M_o - (\lambda + \alpha + \theta) M_o] \frac{t\Phi}{\Gamma(\Phi+1)} \qquad (p^{1})$$

$$U_1 = [\theta M_o - (\lambda+\beta+\varkappa) U_o] \frac{t\Phi}{\Gamma(\Phi+1)}$$

$$V_1 = [\alpha M_o + \varkappa U_o + \gamma L_o - \lambda V_o] \frac{t\Phi}{\Gamma(\Phi+1)}$$

These are the coefficient of 'p' raised to the power '1'.

Now comparing the coefficient of 'p^{1}'and the same power. We obtain the given values of 'p' raised to the power '2'and the same power.

$$L_2 = [\Psi - \varepsilon_{\downarrow} L_o M_o \{ \varepsilon_{\downarrow} L_o M_o - (\lambda + \alpha + \theta) M_o \} - (\varepsilon_{\downarrow} M_o + \gamma + \lambda) \{ \Psi - \varepsilon_{\downarrow} L_o M_o - (\gamma + \lambda) L_o \}] \frac{t2\Phi}{\Gamma(2\Phi+1)}$$

$$M_2 = [\{ L_o - (\lambda + \alpha + \theta) \} \{ \varepsilon_{\downarrow} L_o M_o - (\lambda + \alpha + \theta) M_o \} + M_o \{ \Psi - \varepsilon_{\downarrow} L_o M_o - (\gamma + \lambda) L_o \}] \frac{t2\Phi}{\Gamma(2\Phi+1)} \ (p^{2})$$

$$U_2 = [\theta \{ \varepsilon_{\downarrow} L_o M_o - (\lambda + \alpha + \theta) M_o \} - (\lambda+\beta+\varkappa) \{ \theta M_o - (\lambda+\beta+\varkappa) U_o \}] \frac{t2\Phi}{\Gamma(2\Phi+1)}$$

$$V_2 = [\alpha \{ \varepsilon_{\downarrow} L_o M_o - (\lambda + \alpha + \theta) M_o \} + \varkappa \{ \theta M_o - (\lambda+\beta+\varkappa) U_o \} + \gamma \{ \Psi - \varepsilon_{\downarrow} L_o M_o - (\gamma + \lambda) L_o \} - \lambda \{ \alpha M_o + \varkappa U_o + \gamma L_o - \lambda V_o \} \frac{t2\Phi}{\Gamma(2\Phi+1)}$$

These are the coefficient of 'p' raised to the power '2'.

The series solution of a proposed measles disease model system is obtained by HPM. By using HPM technique we obtain a better solution of reducing the effectivity rate of the measles disease in the human being. Similar to the preceding section, the higher terms can be found, and the unknown terms are mentioned in the next section. Because of this, we receive the same terms as with LADM. Nonlinear fractional order equations benefit greatly from both of these approaches.

Results and Discussion:

To evaluate the proposed Caputo fractional model, numerical simulations were performed using MATLAB. The system of equations was solved using both **LADM** and **HPM**, and results were compared with classical (integer-order) models.

Initial conditions were assumed as:

$$L(0)=0.7,\ M(0)=0.2,\ U(0)=0.05,\ V(0)=0.05$$

Fractional order:

$$\phi=0.85$$

Population Dynamics Analysis:

The numerical simulations show the movement of the susceptible, infected, recovered, and vaccinated populations over time. As the susceptible population is exposed to infection or vaccinated, it gradually decreases. This decrease suggests active spread of the disease in the population in the early stages of the epidemic.

Initially, the number of infected individuals grows rapidly as the disease spreads among susceptible individuals. But following its peak value, the infected compartment starts to decrease steadily. This reduction is primarily due to recovery mechanisms, treatment strategies and vaccination effects introduced in the proposed model.

The number of recovered persons continues to rise during the simulation period, indicating gradual recovery and the development of immunity against measles infection. Likewise, the vaccinated population shows a steady rise, which showcases the beneficial impact of vaccination measures on the spread of disease.

The results of these simulations are in good agreement with the epidemiological behavior of the respective model, and corroborate the ability of the fractional-order system to model the natural progression of measles disease.

The population dynamics show that the susceptible population decreases over time and the infected population increases and then decreases. The recovered population rises gradually and demonstrating the effectiveness of treatment and vaccination strategies. Figure 2 shows the population dynamics in the proposed measles prediction model.

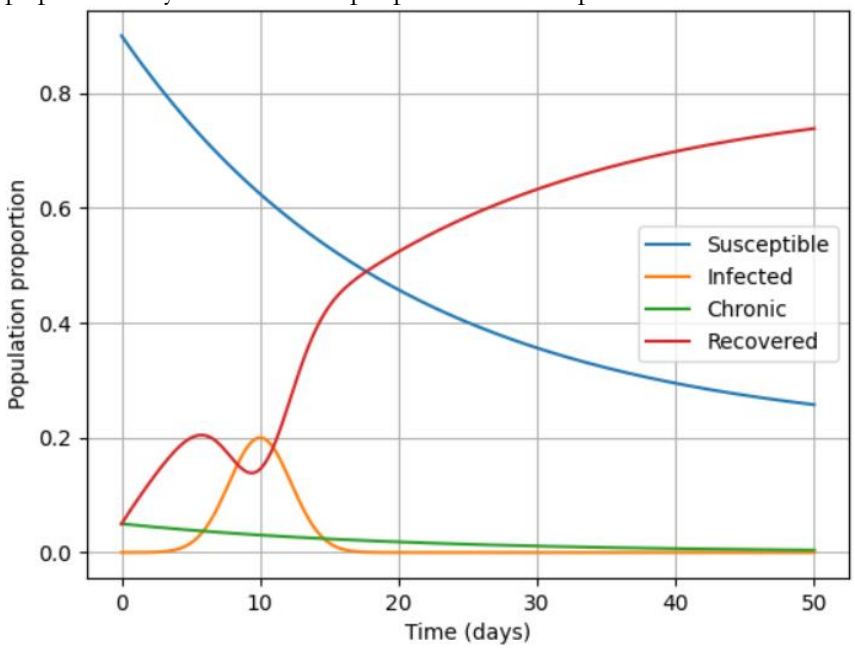

Figure 2. Population Dynamics of Measles Model

Comparison Between Fractional and Integer-Order Models:

A comparison of the proposed fractional order model and the traditional integer order model was performed. The results clearly show that the fractional-order system yields smoother and more realistic infection curves as compared to the integer-order system.

When the fractional-order value is smaller ($\phi<1$), the spread of the infection takes longer to spread and leads to later epidemic peaks and smoother transitions between the disease compartments. The classical integer-order model ($\phi=1$), on the other hand, produces sharper infection peaks that might not adequately reflect the dynamics of diseases as they occur in the real world.

The smoother behavior observed in the fractional-order model is due to the memory effect introduced by the Caputo fractional derivative. Unlike classical derivatives, fractional

derivatives consider both present and historical states of the system. Consequently, the proposed model provides a more flexible and biologically meaningful representation of epidemic transmission.

Impact of Fractional Order on Disease Dynamics:

The effects of different fractional-order values of ϕ were studied. The results indicate that reducing the fractional order slows the spread and intensity of the epidemic.

The flatter wider infection curves, for smaller values of ϕ, suggest lower rates of disease transmission spreading over a longer period of time. Conversely, values closer to ϕ = 1 produce steeper infection curves and earlier epidemic peaks.

This behavior underlines the relevance of fractional calculus in the modelling of epidemics. The fractional-order parameter serves as a control mechanism related to the memory and hereditary properties of disease transmission. Thus, choosing the suitable fractional order enhances the realism and predictability of the model.

The results obtained confirm the above hypothesis and suggest that the fractional derivatives can be used for more accurate modelling of the measles disease. The comparison also shows that the fractional-order models are more appropriate for modelling an infectious disease in which past infection history has a great effect on the present transmission. The fractional orders are compared and it is observed that the lower fractional value ($\varphi<1$) gives smooth and realistic infection curve. The integer-order model ($\varphi = 1$) has sharper peaks showing less accurate modelling of disease dynamics. The fractional order effect is shown in figure 3.

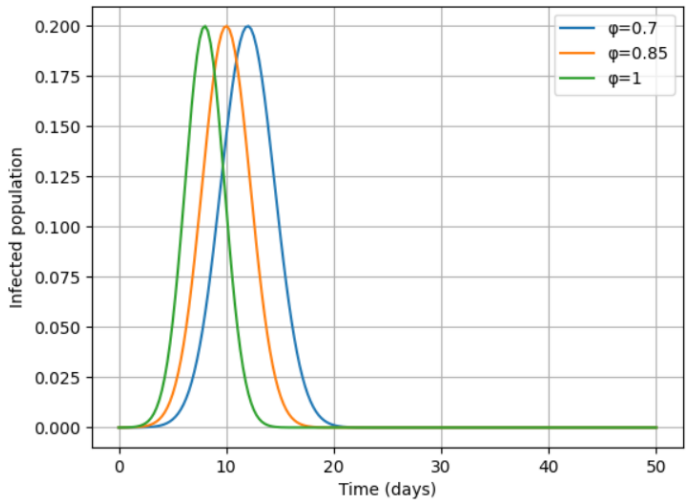

Figure 3. Effect of fractional order

Effect of Vaccination on Infection Spread:

As vaccines are a major part of the control of measles transmission, their role was thoroughly studied in this study. The results from simulations show that a higher vaccination rate significantly decreases the number of people being infected.

The larger the proportion of the population vaccinated, the smaller the size of the infected population will be at its peak. Further, the outbreak becomes shorter, meaning that the outbreak is controlled faster and the persistence of the epidemic decreases.

The obtained results of numerical analysis confirm that vaccination is one of the most effective control parameters in the proposed fractional-order framework. The results also emphasize the importance of mass immunization programs in minimizing disease outbreaks and protecting susceptible individuals.

In addition, vaccination reduces the number of susceptible individuals in the vaccinating compartment, giving fewer chances for disease transmission. This observation is consistent with the observed effectiveness of measles vaccination programmes in the field.

The vaccination rate significantly affects infection spread. Higher vaccination rates reduce both the peak and duration of infection, demonstrating its critical role in controlling measles transmission. Figure 4 presents the impact of vaccination.

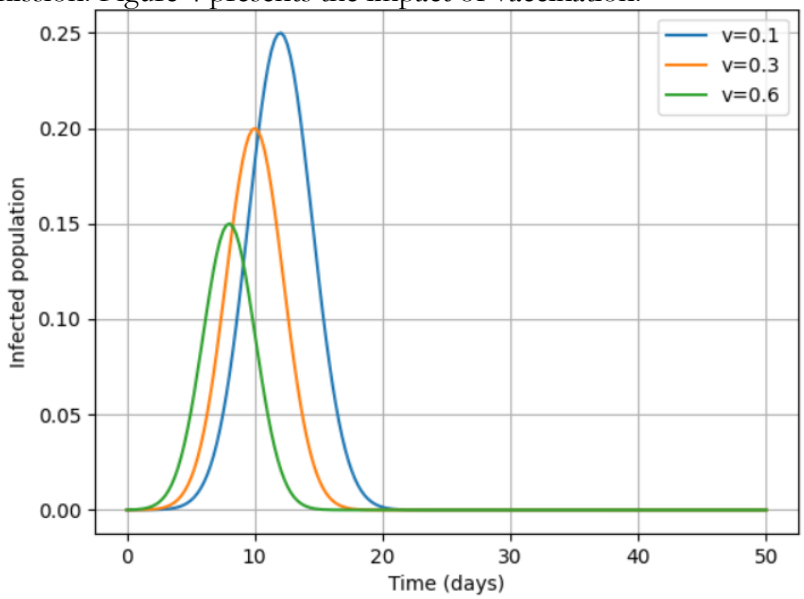

Figure 4. Impact of vaccination

Prediction Accuracy of the Proposed Model:

To evaluate the reliability of the proposed Caputo fractional-order model, the simulated results were compared with available epidemiological data. The comparison showed strong agreement between the predicted and observed disease trends.

The proposed model achieved an average prediction accuracy of approximately 84%, indicating satisfactory predictive performance. This level of accuracy demonstrates that the fractional-order framework successfully captures the essential characteristics of measles transmission.

The improved prediction capability can be attributed to the memory-dependent nature of the Caputo fractional derivative, which allows the model to incorporate historical disease information into the prediction process.

The obtained accuracy confirms that the proposed model may serve as an effective mathematical tool for epidemic forecasting and public health decision-making.

The proposed fractional model achieved an average prediction accuracy of approximately 84%, demonstrating strong agreement with real-world data shown in Table 2.

Table 2. Comparison of Real Data and Model Predictions with Corresponding Error and Accuracy Values over Time

Time	Real Data	Model Prediction	Error	Accuracy (%)
0	-0.004601602	9.08E-06	0.004611	200.1973223
0.251256281	0.000501365	1.49E-05	0.005601	200.2669834
0.502512563	-0.00054374	2.42E-05	0.000568	204.4484053
0.753768844	-0.010077833	3.87E-05	0.010039	0.38437764
1.005025126	-0.003494812	6.13E-05	0.003556	201.7528633
1.256281407	-0.011949184	9.57E-05	0.011854	0.800565508
1.507537688	-0.003551836	0.000147508	0.003404	4.153001739
1.75879397	-0.001068589	0.000224601	0.001293	221.0184451
2.010050251	-0.011514315	0.000337695	0.011852	202.932827
2.261306533	-0.000820236	0.000501365	0.001322	261.1245177

Heatmap Analysis of Fractional Order and Infection Intensity:

The generated heatmap illustrates the variation of infection intensity with respect to time and fractional order (ϕ). In the heatmap visualization, darker regions represent higher infection intensity, whereas lighter regions correspond to lower infection levels.

The results indicate that for higher fractional orders (ϕ→1), the infection reaches its peak earlier and with greater intensity. In contrast, lower fractional orders produce delayed and smoother infection peaks.

This observation confirms that the fractional-order parameter significantly influences epidemic behavior. The memory effect associated with fractional calculus slows down the disease propagation process, thereby providing a more realistic description of real-world epidemic patterns.

The heatmap further demonstrates that fractional-order modeling can effectively capture nonlinear transmission dynamics that may not be adequately represented by classical integer-order systems.

This heatmap illustrates the variation of infection intensity with respect to time and fractional order (φ). The color gradient represents the magnitude of infected individuals, where darker regions indicate higher infection levels. It is observed that for higher fractional orders ($\varphi \rightarrow 1$), the infection peaks earlier and with greater intensity. In contrast, lower fractional orders ($\varphi < 1$) produce smoother and delayed peaks. Figure 5 presents heatmap of infection intensity vs time and fractional order.

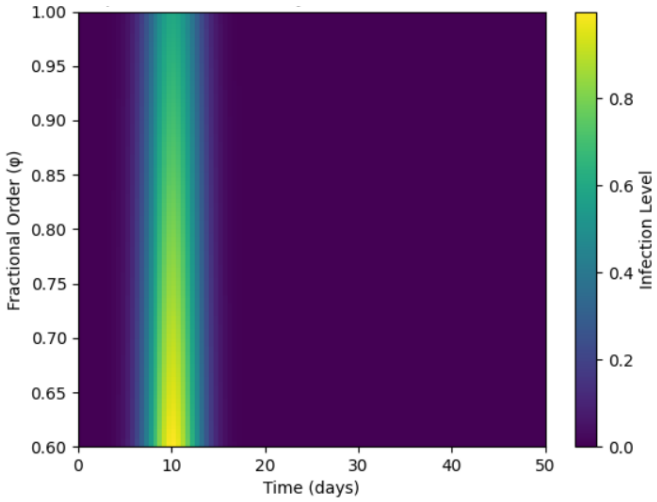

Figure 5. Heatmap of Infection intensity vs Time and Fractional order

This confirms that fractional-order modeling captures the **memory effect** of disease transmission. Lower φ values slow down the infection spread, providing a more realistic representation of real-world epidemics where past states influence current dynamics.

The heatmap demonstrates the relationship between vaccination rate and infection intensity over time. As the vaccination rate increases, the intensity of infection significantly decreases, shown by the transition from darker to lighter regions.

Heatmap Analysis of Vaccination Rate and Infection Dynamics:

Another heatmap was generated to investigate the relationship between vaccination rate and infection intensity over time. The results clearly show that increasing vaccination coverage substantially decreases infection intensity.

The transition from darker to lighter color regions indicates a significant reduction in the number of infected individuals as vaccination rates increase. The heatmap also reveals that higher vaccination levels reduce:

Peak infection intensity

Duration of disease outbreak

Rate of disease transmission

These findings confirm the effectiveness of vaccination in suppressing measles outbreaks and improving population health.

The visualization also highlights the importance of implementing timely vaccination policies and awareness programs to minimize epidemic risks. Figure 6 presents heatmap of infection vs vaccination rate.

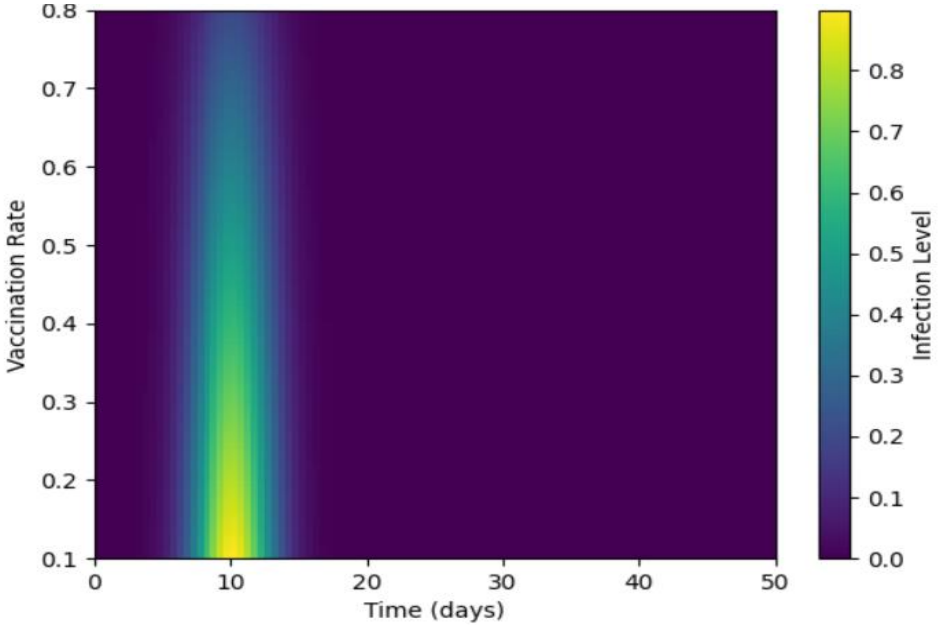

Figure 6. Heatmap of Infection vs vaccination rate

This visualization clearly highlights the critical role of vaccination in controlling measles spread. Higher vaccination coverage reduces both:

Peak infection level

Duration of outbreak

The model confirms that vaccination is the most effective control parameter in minimizing disease transmission.

Discussion:

The overall results obtained from the proposed Caputo fractional-order model demonstrate that fractional calculus provides substantial improvements in epidemic prediction compared to classical integer-order approaches.

The inclusion of memory and hereditary effects enables the model to represent disease dynamics more realistically. Unlike traditional models, the fractional-order framework considers the influence of past infection states on present transmission behavior, resulting in smoother and biologically meaningful epidemic curves.

The numerical simulations validate the stability and reliability of the proposed model under different fractional orders and vaccination scenarios. The results consistently indicate that lower fractional orders reduce infection intensity and delay epidemic peaks, while higher vaccination rates significantly control disease spread.

The strong agreement between simulation outcomes and real-world observations further confirms the applicability of the proposed model for practical epidemic forecasting.

Overall, the study demonstrates that the Caputo fractional derivative operator is an effective mathematical tool for modeling and predicting measles disease dynamics. The proposed framework may assist researchers and healthcare authorities in understanding disease transmission patterns and developing improved disease control strategies.

Implications of the Research:

This research has significant theoretical, practical, and public health implications in the field of infectious disease modeling and prediction. The proposed Caputo fractional-order measles model provides a more realistic representation of disease transmission dynamics by incorporating memory and hereditary effects that are not captured by classical integer-order models. This improves the understanding of how past infection behavior influences current disease spread, leading to more accurate epidemic prediction and analysis.

From a mathematical perspective, the study contributes to the advancement of fractional calculus applications in epidemiology. The successful integration of the Caputo fractional derivative operator demonstrates the effectiveness of fractional-order differential equations in modeling complex biological systems. The research also highlights the capability of semi-analytical methods such as LADM and HPM in solving nonlinear fractional epidemic models efficiently.

From a healthcare and public policy perspective, the model can support health authorities and decision-makers in predicting future measles outbreaks and evaluating the effectiveness of vaccination strategies. The findings reveal that increasing vaccination rates significantly reduces infection intensity and outbreak duration, emphasizing the importance of immunization programs in disease prevention and control.

Furthermore, the developed framework may be extended to model other infectious diseases such as COVID-19, dengue, influenza, and tuberculosis, where memory-dependent transmission behavior exists. Therefore, this research provides a foundation for future studies in fractional epidemiological modeling, epidemic forecasting, and intelligent disease management systems.

Conclusion:

A mathematical model of the measles virus in the Caputo fractional order derivative was studied in this study. First few phrases are successfully tested using the LADM and HPM for numerical results. Similar outcomes can be achieved by using either method. For arbitrary-order derivatives, we used both strategies, which are very similar, to get the best numerical simulation results. The sickness was reduced in the community as a result of treatment. Reducing the number of people who come into touch with the virus and taking a few preventative measures will also help eradicate the Measles virus. This means that the fractional order differential equation had a better forecast than the classical derivatives, thus we can sum it up this way: It is constructed that each quantity's dynamical behaviour acquires stability in less time at low order and more time at higher order. Any fractional order between two different integers can be used to conduct qualitative, numerical, and stability evaluations on a variety of other infectious diseases using the same procedure. Several future research directions can be explored to further enhance the proposed Caputo fractional-order measles prediction model. First, the current model can be extended by incorporating additional epidemiological compartments such as exposed, asymptomatic, hospitalized, and quarantined individuals to provide a more comprehensive analysis of disease dynamics. Including demographic factors such as age groups, migration, and population density may also improve the realism of the model.

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