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Possibility q-Rung Orthopair Fuzzy Soft Framework: An Application for Selection of a Sketcher by Law Enforcement Agency

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The possibility q-rung orthopair fuzzy soft set (Pq-ROFSS) theory is one of effective generalization of possibility Pythagorean fuzzy soft set, possibility intuitionistic fuzzy soft set and possibility fuzzy soft set theories for dealing with the imprecisions and ambiguities in data. The purpose of this paper is to apply this theory in decision making. To achieve this purpose, we first propose the concept of possibility q-rung orthopair fuzzy soft set and define some of its related operations such as union, intersection, complement, De Morgan's law, "AND" and "OR" operations. Further, a new type of similarity measure in possibility q-rung orthopair fuzzy soft environment is defined to deal with the decision making problem. Finally,

Keywords: q-rung orthopair fuzzy soft set; Possibility q-rung orthopair fuzzy soft set; Decision making analysis; Similarity measure.

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we illustrate a numerical example for the selection of a sketcher by law enforcement agencies for the validity of our proposed approach.

Introduction

Selection of a sketcher by law enforcement agencies is one of the most important decision-making problems. Decision making analysis is significant need for a personal or collective. The analysis becomes increasingly complex with the increasing of uncertainties and vagueness in input data coming either from individuals or from some institution. In such problems, the basic issue is to design a balanced tool to choose one object over the other.

In real life problems vagueness can be seen everywhere. Since several decades, researchers have been working out different techniques and methods to deal with data uncertainties. The first remarkable step in this direction can be traced back in 1965 when Zadeh [38] initiated the concept of a fuzzy set and applied this concept successfully to overcome the flaws in a mathematical model of separating useable units from downstate units. After this successful idea, Atanassov [5] developed the theory intuitionistic fuzzy set. Some of its details are in [1, 8, 14]. In the existing work, such limitations that were occurred like membership function for each specific object, lake of parameterization under consideration according to the current situation. In this type of situation, it does not give the right decision for decision-maker. To overcome these limitations, Molodtsov [24] presented the idea of a soft set to deal with the data where uncertainty is due to the inadequacy of involved parameters and also defined fuzzy soft set and intuitionistic fuzzy soft set theory by extended his idea in [21, 20]. Some new operations for intuitionistic fuzzy soft sets also defined in [7]. Some researchers are devoting their attentions on this theory in decision making problems using several techniques. One most reliable method is use of some aggregation operators; these are very beneficial in decision making processes. In (2006) Xu and Yager [32] proposed some aggregation operators to aggregate the data in decision making process. Later on, many researchers gave their concepts to aggregate data by different approaches for dealing with uncertainty [3, 4, 9, 11, 12, 13, 16, 19, 26, 29, 30, 33, 39].

An aggregation operator has been widely used by scientists in decision making processes but in most cases its application on the attributes of the objects, not on the interrelationship of data items. There are several real-life problems where interrelationship between the different objects becomes dynamic, for instance when a decision-maker makes a decision based on thoughts of life risk and cost in a project, he should assign a higher importance to risk than cost. To handle this type of situations, Yager [35] defined Bonferroni mean operator and its generalizations. Afterward the researchers extend Bonferroni mean operators in different tactics [10, 18, 27, 34, 40] and Maclurain symmetric mean operator [28]. The idea of the intuitionistic fuzzy and intuitionistic fuzzy soft set theory is not enough for dealing each kind of problems in decision making process. For instance, when the sum of membership and non-memberships degree is greater than 1, under this type of circumstances decision assessment cannot successfully expressed because in some cases the provided data for a specific attribute be given as in which the sum of membership degree and non-membership degree greater than 1. To deal this type of problems Yager [36] introduced the concept of the Pythagorean fuzzy set and Pythagorean fuzzy soft set defined by Xindong [31]. Yager [37] presented the idea of q-rung orthopair fuzzy set. According to this idea the sum of qth power of both membership and non-membership degree is less equal to 1.

In decision making process, similarity measure is a useful tool for preference one object over the others. According to it, we find the degree of similarity; the larger one would be most preferable. Many researchers gave their ideas to define the similarity measure in different approaches, for instance, Majumdar [23] defined similarity in soft sets also other ideas for similarity measure are given by different researchers as similarity measure in fuzzy soft sets [22], similarity measure in intuitionistic fuzzy sets [17] and similarity measure in intuitionistic fuzzy soft sets discussed in [25]. Further, Alkhazaleh [2] proposed the idea of possibility fuzzy soft set, based on this idea an extension was made by Bashir [6] in the form of possibility intuitionistic fuzzy soft set. After this successful idea, Jia [15] extend this into possibility Pythagorean fuzzy soft set. But the existing ideas are not enough for dealing all kind of problems such as where uncertainty occurs in such a way the sum of membership and non-memberships become greater than 1. Based on this type of problems we propose the idea of possibility q-rung orthopair fuzzy soft set to solve such complex problems in decision making. This paper is planned as follows. We recall some basic ideas in section 2. In section 3, we present the idea of possibility q-rung orthopair fuzzy soft set with some of its basic operations and properties. In order to establish the preference order between two possibility q-rung orthopair fuzzy soft sets a new type of similarity measure is defined in section 4. In section 5 based on our proposed idea the selection criteria have been given in details with the help of a numerical example for the validity of our proposed approach. At the end conclusions are given.

Preliminaries

Definition 2.1: ([24]) Let E be the set of parameters, $A \subset E$ and U a universal set. A soft set can be identified by a pair(F, A), provided that $F : A \longrightarrow \mathrm{P}^U$, where P^U is set of all subsets of U.

Definition 2.2: ([21]) Let E be the set of parameters, $A \subset E$ and U a universal set. The pair (F, A) is called fuzzy soft set, given that $F: A \longrightarrow F^U$, where F^U is a set of all fuzzy subsets of U.

Definition 2.3: ([20]) Let E be the set of parameters, $A \subset E$ and U a universal set. The pair (F, A) is called an intuitionistic fuzzy soft set, if $F: A \longrightarrow \mathrm{IF}^U$, where IF^U denotes the set of all intuitionistic fuzzy subsets of U.

Definition 2.4: ([36]) Let X be a universe, a Pythagorean fuzzy set is defined on as follows:

$F(x_i) = \{ \langle x_i, \ \mu_F(x_i), \ \nu_F(x_i) \rangle : x_i \in X\}$, where $\mu_k : X \longrightarrow [0, 1]$, and $\nu_k : X \longrightarrow [0, 1]$ are membership and non-memberships degrees of the elements of X with the condition $0 \leq \mu_k(x_i)^2 + \nu_k(x_i)^2 \leq 1$. The degree of hesitancy is given by $I_k(x_i) = (1 - \mu_k(x_i)^2 + \nu_k(x_i)^2)^{\frac{1}{2}}$.

Definition 2.5: ([31]) Let X be a universe, E a set of parameters. If PFS^U denotes the set of all Pythagorean fuzzy sets of X and $A \subset E$. Then a pair (F, A) is called Pythagorean fuzzy soft set (PFSS) provided that $F : A \longrightarrow \mathrm{PFS}^U$. This means that for any $e_k \in A$, the PFS is identified as

$F_{e_k}(x_i) = \{\langle x_i,\ \mu_{e_k}(x_i),\ \nu_{e_k}(x_i)\rangle : x_i \in X\}$, where $\mu_k : X \longrightarrow [0,1]$, and $\nu_k : X \longrightarrow [0,1]$ are membership and non-memberships degrees of the elements of X with the condition that $0 \leq \mu_k(x_i)^2 + \nu_k(x_i)^2 \leq 1$. The degree of hesitancy is given by $I_k(x_i) = (1 - \mu_k(x_i)^2 + \nu_k(x_i)^2)^{\frac{1}{2}}$.

Definition 2.6: ([37]) Let X be a universe, the q-rung orthopair fuzzy set (q-ROFS) is defined as $F_k(x_i) = \{\langle x_i,\ \mu_k(x_i),\ \nu_k(x_i)\rangle : x_i \in X\}$, where $\mu_k : X \longrightarrow [0,1]$, and $\nu_k : X \longrightarrow [0,1]$ are membership and non-memberships degrees of elements of X with the condition that $0 \leq \mu_k(x_i)^q + \nu_k(x_i)^q \leq 1,\ (q \geq 1)$. The degree of hesitancy is given as $I_k(x_i) = (\mu_k(x_i)^q + \nu_k(x_i)^q - \mu_k(x_i)^q \nu_k(x_i)^q)^{\frac{1}{q}}$.

Definition 2.7: ([2]) Let U be a universe, E a set of parameters, the pair (U, E) is called soft universe. Suppose that $F : E \longrightarrow \mathrm{I}^U$ and $\mu : E \longrightarrow \mathrm{I}^U$ a given mapping, where I^U is collection of all fuzzy subsets of U. Let the function $F_\mu : E \longrightarrow \mathrm{I}^U \times \mathrm{I}^U$ be defined as follows: $F_\mu(e) = \{\langle x,\ F(e)(x),\ \mu(e)(x)\rangle : x \in U\}$, then F_μ is called possibility fuzzy soft set over the soft universe (U, E).

Definition 2.8: ([6]) Let U be a universe, E a set of parameters and (U, E) a soft universe. Suppose that $F: E \longrightarrow (\mathrm{I} \times \mathrm{I})^U \times \mathrm{I}^U$, where $(\mathrm{I} \times \mathrm{I})^U$ denotes the collection of all intuitionistic fuzzy subsets of, I^U denotes the collection of all fuzzy subset of U *and* $\mu : E \longrightarrow \mathrm{I}^U$. Let the function $F_\mu : E \longrightarrow (\mathrm{I} \times \mathrm{I})^U \times \mathrm{I}^U$ defined as follows: $F_\mu(e) = \{\langle x,\ F(e)(x),\ \mu(e)(x)\rangle : x \in U\}$, then F_μ is called possibility intuitionistic fuzzy soft set over the soft universe (U, E).

Definition 2.9: ([15]) Let (U, E) be a soft universe, $F : E \longrightarrow PF^U$ where PF^U collection of all Pythagorean fuzzy subsets of U, and $\mu : E \longrightarrow PF^U$. Let us define $F_\mu : E \longrightarrow PF^U \times PF^U$ by $F_\mu(e) = \{\langle x,\ F(e)(x),\ \mu(e)(x)\rangle : x \in U\}$, then F_μ is called possibility Pythagorean fuzzy soft set over the soft universe (U, E).

Definition 2.10: Let (U, E) be a soft universe, and $F : A \longrightarrow \mathrm{q-ROFS}^U$, where $\mathrm{q-ROFS}^U$ denotes the set of all q-rung orthopair fuzzy sets (q-ROFS) of X and $A \subset E$. The pair (F, A) is called q-rung orthopair fuzzy soft set (q-ROFSS). This means that for any $e_k \in A$ the q-ROFSS is identified as $F_{e_k}(x_i) = \{\langle x_i,\ \mu_k(x_i),\ \nu_k(x_i)\rangle : x_i \in X\}$, where $\mu_k : X \longrightarrow [0,1]$, and $\nu_k : X \longrightarrow [0,1]$ are membership and non-memberships degrees of the elements of X with the condition that $0 \leq \mu_k(x_i)^q + \nu_k(x_i)^q \leq 1,\ (q \geq 1)$. The degree of hesitancy is given as $I_k(x_i) = (\mu_k(x_i)^q + \nu_k(x_i)^q - \mu_k(x_i)^q \nu_k(x_i)^q)^{\frac{1}{q}}$.

Possibility q-Rung Orthopair Fuzzy Soft set

In this section, we introduce the concept of possibility q-rung orthopair fuzzy soft set (Pq-ROFSS) to extend the concept of possibility Pythagorean fuzzy soft set.

Definition 3.1: Let (U, E) be a soft universe, $F : E \longrightarrow \mathrm{q-ROFS}^U$ where $\mathrm{q-ROFS}^U$ denotes the set of all q-rung orthopair fuzzy sets (q-ROFSs) of U and $\varphi : E \longrightarrow \mathrm{q-ROFS}^U$, then the mapping $F_\varphi : E \longrightarrow \mathrm{q-ROFS}^U \times \mathrm{q-ROFS}^U$ defined by $F_\varphi(e) = \{ \langle x, F(e)(x), \varphi(e)(x) \rangle : x \in U\}$ is called possibility q-rung orthopair fuzzy soft set (Pq-ROFSS) on (U, E). We may write $F_\varphi(e)$ as $F_\varphi(e) = \{ \left(\mu_{F(e)}(x), \nu_{F(e)}(x)\right), \left(\mu_{\varphi(e)}(x), \nu_{\varphi(e)}(x)\right) : x \in U\}$.

For simplicity we write $F_\varphi = (\mu_F, \nu_F), (\mu_\varphi, \nu_\varphi)$, where $\mu_F : U \longrightarrow [0, 1]$, and $\nu_F : U \longrightarrow [0, 1]$ are membership and non-memberships degrees of the elements of U with the condition that $0 \leq \mu_F(x)^q + \nu_F(x)^q \leq 1$, $(q \geq 1)$. The degree of hesitancy is given as $I_F(x) = (\mu_F(x)^q + \nu_F(x)^q - \mu_F(x)^q \nu_F(x)^q)^{\frac{1}{q}}$ and we have the similar representations for μ_φ and ν_φ.

Example 1: $H = \{h_1, h_2, h_3\}$ be set of three houses under consideration for purchase and $E = \{e_1 = \text{cost}, e_2 = \text{location}, e_3 = \text{area}\}$ a set of parameters. Suppose the Pq-ROFSS is given as follows

$$F_\varphi(e_1) = \{h_1/(0.8, 0.7), (0.7,0.6), h_2/(0.9,0.7), (0.8,0.7), h_3/(0.5,0.5), (0.5,0.4)\}$$

$$F_\varphi(e_2) = \{h_1/(0.9, 0.5), (0.8,0.4), h_2/(0.6,0.5), (0.7,0.4), h_3/(0.5,0.6), (0.7,0.6)\}$$

$$F_\varphi(e_3) = \{h_1/(0.7, 0.9), (0.2,0.5), h_2/(0.5,0.7), (0.4,0.3), h_3/(0.6,0.7), (0.5,0.3)\}$$

Now the representation of Pq-ROFSS described above can be written in matrix form as follows

$$F_\varphi = \begin{bmatrix} (0.8, 0.7), (0.7,0.6) & (0.9,0.7), (0.8,0.7) & (0.5,0.5), (0.5,0.4) \\ (0.9, 0.5), (0.8,0.4) & (0.6,0.5), (0.7,0.4) & (0.5,0.6), (0.7,0.6) \\ (0.7, 0.9), (0.2,0.5) & (0.5,0.7), (0.4,0.3) & (0.6,0.7), (0.5,0.3) \end{bmatrix}$$

Note 1: Note that in the above example, the first sample h_1 with respect to e_1 has a membership degree 0.8 and non-membership degree 0.7 but the sum of their squares is greater than 1. This case cannot be dealt with possibility Pythagorean fuzzy soft set theory defined by Jia in [15]. Moreover, we have the following observations:

i. For q=2, the Definition (3.1) reduces to possibility Pythagorean fuzzy soft set given by Jia [15].
ii. For q=1 and $\nu_\varphi(e_i) = 0$ for all i, the Definition (3.1) reduces to possibility intuitionistic fuzzy soft set given by Bashir in [6].

iii. For q=1, $\nu_F(e_i) = 0$ and $\nu_\varphi(e_i) = 0$ for all i, the Definition (3.1) reduces to possibility fuzzy soft set proposed by Alkhazaleh in [2].

Definition 3.2: Let (U, E) be soft universe, F_φ and G_ψ be two Pq-ROFSSs over (U, E). We say F_φ as possibility q-rung orthopair fuzzy soft subset of G_ψ, if and only if the follows hold

i. $\varphi(e)(x) \subseteq \psi(e)(x)$.
ii. $F(e)(x) \subseteq G(e)(x)$.

We denote it as $F_\varphi \subseteq G_\psi$, where (i) means that $\mu_\varphi \leq \mu_\psi$ and $\nu_\varphi \geq \nu_\psi$ and (ii) gives $\mu_F \leq \mu_G$ and $\nu_F \geq \nu_G$.

Example 2: Consider the soft universe (H, E) given in Example (1). Let G_ψ be another the Pq-ROFSS given as follows

$$G_\psi\,(e_1) = \{h_1/(0.6, 0.8), (0.5,0.8), h_2/(0.4,0.7), (0.6,0.8), h_3/(0.4,0.6), (0.4,0.8)\}$$

$$G_\psi\,(e_2) = \{h_1/(0.6, 0.7), (0.4,0.7), h_2/(0.3,0.7), (0.4,0.4), h_3/(0.4,0.7), (0.6,0.8)\}$$

$$G_\psi\,(e_3) = \{h_1/(0.5, 0.9), (0.1,0.7), h_2/(0.4,0.9), (0.3,0.6), h_3/(0.5,0.9), (0.2,0.7)\}$$

Clearly, one can see that $G_\psi \subseteq F_\varphi$.

Definition 3.3: Let (U, E) be soft universe, F_φ and G_ψ be two Pq-ROFSSs over(U, E). We say F_φ as possibility q-rung orthopair fuzzy soft equal of G_ψ, if and only if $G_\psi \subseteq F_\varphi$ and $F_\varphi \subseteq G_\psi$. Then we can say $F_\varphi = G_\psi$.

Operations on possibility q-rung orthopair fuzzy soft

Definition 3.4: Let (U, E) be soft universe and F_φ be a Pq-ROFSS over (U, E). The complement of F_φ is denoted by ${F_\varphi}^c$ and is defined as ${F_\varphi}^c = \left(F^C(e)(x), \varphi^C(e)(x)\right)$, where $F^C(e)(x) = \left(\nu_{F(e)}(x), \mu_{F(e)}(x)\right)$ and $\varphi^C(e)(x) = \left(\nu_{\varphi(e)}(x), \mu_{\varphi(e)}(x)\right)$. From this definition, note that $({F_\varphi}^c)^c = F_\varphi$.

Definition 3.5: Let (U, E) be soft universe, F_φ and G_ψ be two Pq-ROFSSs over(U, E). Then the union and intersection of two Pq-ROFSSs F_φ and G_ψ over (U, E) are denoted by $F_\varphi \cup G_\psi$ and $F_\varphi \cap G_\psi$, respectively and is defined as

$$F_\varphi \cup G_\psi = \Big\{\Big(\max_i\{\mu_{F(e_i)}(x_i), \mu_{G(e_i)}(x_i)\}, \quad \min_i\{\nu_{F(e_i)}(x_i), \nu_{G(e_i)}(x_i)\}\Big), \\ \Big(\max_i\{\mu_{\varphi(e_i)}(x_i), \mu_{\psi(e_i)}(x_i)\}, \quad \min_i\{\nu_{\varphi(e_i)}(x_i), \nu_{\psi(e_i)}(x_i)\}\Big)\Big\}$$

And

$$F_\varphi \cap G_\psi = \left\{\left(\min_i\{\mu_{F(e_i)}(x_i), \mu_{G(e_i)}(x_i)\}, \quad \max_i\{\nu_{F(e_i)}(x_i), \nu_{G(e_i)}(x_i)\}\right), \left(\min_i\{\mu_{\varphi(e_i)}(x_i), \mu_{\psi(e_i)}(x_i)\}, \quad \max_i\{\nu_{\varphi(e_i)}(x_i), \nu_{\psi(e_i)}(x_i)\}\right)\right\}.$$

Example 3: Suppose that F_φ and G_ψ are two Pq-ROFSSs over (U, E) defined as follows

$$F_\varphi(e_1) = \{u_1/(0.5, 0.7), (0.6,0.4), u_2/(0.2,0.7), (0.8,0.5), u_3/(0.3,0.5), (0.6,0.3)\}$$

$$F_\varphi(e_2) = \{u_1/(0.6, 0.5), (0.7,0.4), u_2/(0.5,0.2), (0.6,0.6), u_3/(0.5,0.3), (0.9,0.6)\}$$

$$F_\varphi(e_3) = \{u_1/(0.5, 0.4), (0.7,0.4), u_2/(0.6,0.4), (0.6,0.3), u_3/(0.8,0.7), (0.4,0.3)\}$$

And

$$G_\psi\ (e_1) = \{u_1/(0.6, 0.3), (0.7,0.4), u_2/(0.6,0.7), (0.6,0.3), u_3/(0.5,0.4), (0.7,0.5)\}$$

$$G_\psi\ (e_2) = \{u_1/(0.7, 0.3), (0.4,0.6), u_2/(0.3,0.5), (0.7,0.4), u_3/(0.7,0.7), (0.4,0.6)\}$$

$$G_\psi\ (e_3) = \{u_1/(0.6, 0.9), (0.6,0.5), u_2/(0.5,0.7), (0.4,0.6), u_3/(05,0.5), (0.7,0.4)\}$$

Then the union and intersection given in matrix form are given as follows

$$F_\varphi \cup G_\psi = \begin{bmatrix} (0.6, 0.3), (0.7,0.4) & (0.6,0.7), (0.8,0.3) & (0.5,0.4), (0.7,0.3) \\ (0.7, 0.3), (0.7,0.4) & (0.5,0.2), (0.7,0.4) & (0.7,0.3), (0.9,0.6) \\ (0.6, 0.4), (0.7,0.4) & (0.6,0.4), (0.6,0.3) & (0.8,0.5), (0.7,0.3) \end{bmatrix}$$

$$F_\varphi \cap G_\psi = \begin{bmatrix} (0.5, 0.7), (0.6,0.4) & (0.2,0.7), (0.6,0.5) & (0.3,0.5), (0.6,0.5) \\ (0.6, 0.5), (0.4,0.6) & (0.3,0.5), (0.6,0.6) & (0.5,0.7), (0.4,0.6) \\ (0.5, 0.9), (0.6,0.5) & (0.5,0.7), (0.4,0.6) & (0.5,0.7), (0.4,0.4) \end{bmatrix}$$

Definition 3.6: Let N_δ be a Pq-ROFSS over (U, E). We say N_δ is possibility null q-rung orthopair fuzzy soft set if $N_\delta : E \longrightarrow \text{q} - \text{ROFS}^U \times \text{q} - \text{ROFS}^U$ is defined as $N_\delta(e) = \langle\, N(e)(x), \delta(e)(x)\rangle$, where $N(e)(x) = (0, 1)$ and $\delta(e)(x) = (0, 1)$ for all $x \in U$.

Definition 3.7: Let S_ε be a Pq-ROFSS over (U, E). We say S_ε is possibility sure q-rung orthopair fuzzy soft set if $S_\varepsilon : E \longrightarrow \text{q} - \text{ROFS}^U \times \text{q} - \text{ROFS}^U$ is defined as $S_\varepsilon(e) = \langle\, S(e)(x),\ \varepsilon(e)(x)\rangle$, where $S(e)(x) = (1, 0)$ and $\varepsilon(e)(x) = (1, 0)$ for all $x \in U$.

Theorem 3.1: For any Pq-ROFSS F_φ over (U, E), the followings hold

1) $F_\varphi = F_\varphi \cup F_\varphi,\ \ F_\varphi = F_\varphi \cap F_\varphi,$
2) $F_\varphi \subseteq F_\varphi \cup F_\varphi,\ \ F_\varphi \subseteq F_\varphi \cap F_\varphi,$

3) $F_{\varphi} \cup N_{\delta} = F_{\varphi}$, $F_{\varphi} \cap N_{\delta} = N_{\delta}$,
4) $F_{\varphi} \cup S_{\varepsilon} = S_{\varepsilon}$, $F_{\varphi} \cap S_{\varepsilon} = F_{\varphi}$.

Proof: The proofs are straightforward in the light of Definitions (3.2) and (3.5).

Note: By the above definitions on can easily check that if $F_{\varphi} \neq S_{\varepsilon}$ or $F_{\varphi} \neq N_{\delta}$ then $F_{\varphi} \cup {F_{\varphi}}^{c} \neq S_{\varepsilon}$ and $F_{\varphi} \cap {F_{\varphi}}^{c} \neq N_{\delta}$

Theorem 3.2: For any Pq-ROFSSs F_{φ}, G_{ψ} and K_{σ} over(U, E), commutative and associative law holds for union and intersection, which are given as:

1) $(F_{\varphi} \cup G_{\psi}) = (G_{\psi} \cup F_{\varphi})$,
2) $(F_{\varphi} \cap G_{\psi}) = (G_{\psi} \cap F_{\varphi})$,
3) $(F_{\varphi} \cup G_{\psi}) \cup K_{\sigma} = F_{\varphi} \cup (G_{\psi} \cup K_{\sigma})$,
4) $(F_{\varphi} \cap G_{\psi}) \cap K_{\sigma} = F_{\varphi} \cap (G_{\psi} \cap K_{\sigma})$.

Theorem 3.3: For any Pq-ROFSSs F_{φ}, G_{ψ} and K_{σ} over (U, E), distributive laws holds

1) $F_{\varphi} \cup (G_{\psi} \cap K_{\sigma}) = (F_{\varphi} \cup G_{\psi}) \cap (F_{\varphi} \cup K_{\sigma})$,
2) $F_{\varphi} \cap (G_{\psi} \cup K_{\sigma}) = (F_{\varphi} \cap G_{\psi}) \cup (F_{\varphi} \cap K_{\sigma})$.

Theorem 3.4: For any Pq-ROFSSs F_{φ} and G_{ψ} over(U, E), we have the following:

1) $(F_{\varphi} \cup G_{\psi})^{c} = {F_{\varphi}}^{c} \cap {G_{\psi}}^{c}$,
2) $(F_{\varphi} \cap G_{\psi})^{c} = {F_{\varphi}}^{c} \cup {G_{\psi}}^{c}$.

Proof: It can be easily verified by Definitions (3.4) and (3.5).

Now we discuss the "OR" and "AND" operation for two Pq-ROFSSs over (U, E).

Definition 3.8: Let (F_{φ}, A) and (G_{ψ}, B) be two Pq-ROFSSs over (U, E), where $A, B \subseteq E$ then the "(F_{φ}, A) OR (G_{ψ}, B)" is denoted by $(F_{\varphi}, A) \vee (G_{\psi}, B)$ and is defined as

$$(F_{\varphi}, A) \vee (G_{\psi}, B) = (F(a) \cup G(b)), (\varphi(a) \cup \psi(b)), \qquad \text{where } a \in A \text{ and } b \in B.$$

Definition 3.9: Let (F_{φ}, A) and (G_{ψ}, B) be two Pq-ROFSSs over (U, E), where $A, B \subseteq E$ then the "(F_{φ}, A) AND (G_{ψ}, B)" is denoted by $(F_{\varphi}, A) \wedge (G_{\psi}, B)$ and is defined as

$$(F_{\varphi}, A) \wedge (G_{\psi}, B) = (F(a) \cap G(b)), (\varphi(a) \cap \psi(b)), \qquad \text{where } a \in A \text{ and } b \in B.$$

Theorem 3.5: For any Pq-ROFSSs F_φ and G_ψ over (U,E), the operations given in the Definitions (3.8) and (3.9) satisfy the following:

1) $((F_\varphi,A) \vee (G_\psi,B))^c = (F_\varphi,A)^c \wedge (G_\psi,B)^c$,
2) $((F_\varphi,A) \wedge (G_\psi,B))^c = (F_\varphi,A)^c \vee (G_\psi,B)^c$.

Proof: Proof of the theorem is straightforward by Definitions (3.8) and (3.9).

Similarity measure between possibility q-rung orthopair fuzzy soft sets

In this section, we introduce the concept of similarity measure between any two Pq-ROFSSs, inspired by the ideas of Muthukumar [25], Garg [13] and Jia [15].

Definition 4.1: Let (U,E) be soft universe, F_φ and G_ψ are two Pq-ROFSSs over(U,E). Then the similarity measure between F_φ and G_ψ is denoted by $Sim(F_\varphi\,,\ G_\psi)$ and is defined as

$$Sim(F_\varphi\,, G_\psi) = \rho(F.G).\kappa(\varphi,\psi) \qquad (1)$$

Such that

$$\rho(F.G) = \frac{M\big(F(e)(x),G(e)(x)\big) + N\big(F(e)(x),G(e)(x)\big)}{2} \qquad (2)$$

And

$$\kappa(\varphi,\psi) = 1 - \frac{\sum_{i=1}^{n}|\theta_i - \omega_i|}{\sum_{i=1}^{n}|\theta_i + \omega_i|} \qquad (3)$$

Where

$$M\big(F(e)(x),G(e)(x)\big) = \frac{\sum_{i=1}^{n}\left(\mu_{F(e_i)}(x)\right)^q\left(\mu_{G(e_i)}(x)\right)^q}{\sum_{i=1}^{n} 1 - \left(1 - \left(\mu_{F(e_i)}(x)\right)^{2q} - \left(\mu_{G(e_i)}(x)\right)^{2q} + \left(\mu_{F(e_i)}(x)\right)^{2q}\left(\mu_{G(e_i)}(x)\right)^{2q}\right)^{\frac{1}{2}}} \qquad (4)$$

$$N\big(F(e)(x),G(e)(x)\big) = \left(1 - \frac{\sum_{i=1}^{n}\left|\left(\nu_{F(e_i)}(x)\right)^q - \left(\nu_{G(e_i)}(x)\right)^q\right|}{\sum_{i=1}^{n}\left(1 - \left(\nu_{F(e_i)}(x)\right)^q.\left(\nu_{G(e_i)}(x)\right)^q\right)}\right)^{\frac{1}{q}} \qquad (5)$$

$$\theta_i = \frac{\left(\mu_{\varphi(e_i)}(x)\right)^q}{\left(\mu_{\varphi(e_i)}(x)\right)^q + \left(\nu_{\varphi(e_i)}(x)\right)^q} \quad \text{and} \quad \omega_i = \frac{\left(\mu_{\psi(e_i)}(x)\right)^q}{\left(\mu_{\psi(e_i)}(x)\right)^q + \left(\nu_{\psi(e_i)}(x)\right)^q}.$$

Example 4: Consider two Pq − ROFSSs, F_φ and G_ψ over (U, E) which are given in the Example. It is given that

$$F_\varphi = \begin{bmatrix} (0.5, 0.7), (0.6, 0.4) \\ (0.6, 0.5), (0.7, 0.4) \\ (0.5, 0.4), (0.7, 0.4) \end{bmatrix} \text{ and } G_\psi = \begin{bmatrix} (0.6, 0.3), (0.7, 0.4) \\ (0.7, 0.3), (0.4, 0.6) \\ (0.6, 0.9), (0.6, 0.5) \end{bmatrix}$$

For $q = 3$, using the Definition (4.1) we have

$$M\left(F(e)(x), G(e)(x)\right) = \frac{0.027 + 0.0740 + 0.027}{0.0312 + 0.0827 + 0.0312} = 0.8815$$

And

$$N\left(F(e)(x), G(e)(x)\right) = \left(1 - \frac{0.316 + 0.098 + 0.665}{0.9907 + 0.9966 + 0.9533}\right)^{\frac{1}{3}} = \left(1 - \frac{1.079}{2.9406}\right)^{\frac{1}{3}} = 0.8586$$

Hence,

$$\rho(F.G) = \frac{0.8815 + 0.8586}{2} = 0.8700$$

Also

$$\kappa(\varphi, \psi) = 1 - \frac{0.0713 + 0.6142 + 0.1507}{1.6141 + 1.0712 + 1.1161} = 1 - \frac{0.8362}{3.8014} = 0.7800$$

Hence, we have

$$Sim\left(F_\varphi, G_\psi\right) = 0.8700 \times 0.7800 = 0.6786$$

Theorem 4.1: For any Pq-ROFSSs F_φ, G_ψ and K_σ over(U, E), by the proposed Definition (4.1), we have following basic properties of similarity measure

1. $Sim\left(F_\varphi, G_\psi\right) \in [0, 1]$,
2. $Sim\left(F_\varphi, G_\psi\right) = Sim\left(G_\psi, F_\varphi\right)$
3. If $F_\varphi \subseteq G_\psi \subseteq K_\sigma$ then $Sim\left(F_\varphi, K_\sigma\right) \leq Sim\left(G_\psi, K_\sigma\right)$
4. If $F_\varphi = G_\psi$ then $Sim\left(F_\varphi, G_\psi\right) = 1$,

5. $Sim(F_{\varphi}, G_{\psi}) = 0$ if and only if $F_{\varphi} \cap G_{\psi} = N_{\delta}$, where N_{δ} defined in the Definition (3.6).

Application in Decision Making Analysis for Selections of a Sketcher

Decision making theory is very useful and important in real-life problems. In every field, decision science has imperative role, like in economy, engineering, medical, management politics and many more. For instance, law enforcement agencies always try to reduce the crime rate in their State. For this purpose they need the sketch of a unknown criminal to identify the criminal and it facilitates the police agencies to get their target and help to arrest the criminal. The rough sketches of the criminal would display on public places and social media for identification. Considerate how humans identify face sketches drawn by sketcher is of weighty value to criminal investigators, intelligence agencies and researchers. However, large scale investigational studies of hand-drawn face and computer made sketches are still very inadequate in terms of the number of sketchers, the number of sketches and the number of human evaluators involved. One main cause of uncertainties is that, in a typical criminal inquiry, sketches were almost always recognized by individuals. The better and fair sketch helps agencies to achieve their target. The notable reason behind the achievements for an agency is that they have a talented sketcher. In the selection of a sketcher by an agency, it is important to select the best one according to various standards of experts is a decision-making problem. Our target is to select the optimal one out of large number of alternatives based on the evaluation of experts against the criteria.

Numerical Study:

A law enforcement agency intends to hire a skillful employ to draw the sketches of suspects. It is desired that the person hired may sketch any image according to different conditions: if the CCTV footage of a crime scene is available but unable to provide a clear identification of a suspect, the person may draw the sketch of suspects based on that video by estimating properly the different poses of the criminals and if CCTV footage is not available then may sketch may be drawn according to comments and suggestions of the peoples those are present on that crime scene. Therefore, the agency needs a brilliant and intelligent sketcher.

An original picture is given in Figure (1) and ideal score of sketcher for figure (1) is given in Table (1), which is indeed a Pq-ROFSS in a tabular form.

Table 1: Pq-ROFSS for an idea artist for figure (1)

$S_{\varepsilon}(e)$	e_1	e_2	e_3	e_4
$S(e)(x)$	$(1,0)$	$(1,0)$	$(1,0)$	$(1,0)$
$\varepsilon(e)(x)$	$(1,0)$	$(1,0)$	$(1,0)$	$(1,0)$

The candidates are asked to present the hand-drawn sketch to the experts and the selection would be done on the basis of their perfection with reference to Table 1. In this selection process, the score of each candidate will be evaluated by experts on the basis of parameters $E = \{e_1 =$ IQ level,$e_2 =$ mind reader, $e_3 =$ highly qualified, $e_4 =$ computer skills$\}$. The set $U = \{X, Y, Z\}$ of three candidates is taken as alternatives. Suppose the decision-makers in an agency get the Pq-ROFSN values for each candidate. The figures (2), (3) and (4) are made by X, Y and Z respectively.

Figure: 1

Figure: 2

Figure: 3

Figure: 4

The evaluations of the sketchers as per Pq-ROFSS are given in Tables 2-4 provided by the experts depending upon their evaluation of alternatives against the criteria under consideration.

Table 2: Pq-ROFSS for ***X*** by figure (2)

$X_\eta(e)$	e_1	e_2	e_3	e_4
$X(e)(x)$	$(0.6, 0.5)$	$(0.7, 0.4)$	$(0.4, 0.5)$	$(0.3, 0.6)$
$\eta(e)(x)$	$(0.4, 0.6)$	$(0.3, 0.8)$	$(0.3, 0.7)$	$(0.5, 0.5)$

Table 3: Pq-ROFSS for ***Y*** by figure (3)

$Y_\xi(e)$	e_1	e_2	e_3	e_4
$Y(e)(x)$	$(0.9, 0.2)$	$(0.5, 0.3)$	$(0.7, 0.2)$	$(0.8, 0.1)$
$\xi(e)(x)$	$(0.7, 0.1)$	$(0.6, 0.4)$	$(0.7, 0.1)$	$(0.8, 0.2)$

Table 4: Pq-ROFSS for Z by figure (4)

$Z_\lambda(e)$	e_1	e_2	e_3	e_4
$Z(e)(x)$	$(0.6, 0.2)$	$(0.6, 0.4)$	$(0.5, 0.4)$	$(0.7, 0.3)$
$\lambda(e)(x)$	$(0.6\,, 0.3)$	$(0.5, 0.4)$	$(0.6, 0.2)$	$(0.7, 0.4)$

To select a sketcher whose score is closed to an ideal value, one needs to find the similarity measure by applying the Definition (4.1). The similarity measures for all alternatives with the ideal score values are evaluated for each candidate. The minimum score of similarity with ideal value for qualifying the first round of selection is 0.5.

Here we give the details for calculation of the similarity measure of Y with the ideal score as follows:

For $q = 3$, using the Definition (4.1) we have

$$M\big(S(e)(x), Y(e)(x)\big) = \frac{1.702}{4} = 0.4272$$

And

$$N\big(S(e)(x), Y(e)(x)\big) = 1$$

Hence,

$$\rho(S, Y) = \frac{0.4272 + 1}{2} = 0.7136$$

Also

$$\kappa(\varepsilon, \xi) = 1 - \frac{0.2497}{7.7502} = 0.9677$$

Hence, we have

$$Sim\big(S_\varepsilon\,, Y_\xi\big) = 0.7136 \times 0.9677 = 0.6906$$

Similarly, we find the similarity measure for X with the ideal score is computed as

$$Sim\big(S_\varepsilon\,, X_\eta\big) = 0.2040$$

And the similarity measure for Z with the ideal score is computed as

$$Sim(S_\varepsilon\,, Z_\lambda) = 0.5590.$$

Note that Y is the best candidate among three, as it has highest similarity measure 0.6906 with ideal score. The second preference is for Z and X cannot qualify the selection criteria.

Conclusion

In this paper, the main motivation is to present the notion of possibility q-rung orthopair fuzzy soft set to solve problems in decision making analysis along with considering the possibility of belongingness of the element in the universe. We also defined some of its basic's operations such as union, intersection, complement, De Morgan's law, "AND" and "OR" operations, we also proposed a new type of similarity measure to compare possibility q-rung orthopair fuzzy soft sets for dealing problems. For the validity of our proposed approach we discussed a numerical example in decision making process. In future work, we should try to define some aggregation operators especially Bonferroni mean operator in possibility q-rung orthopair fuzzy soft environment.

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