



Unveiling Worldwide Fossil Fuel Demands and the Dynamics of the Carbon Market

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This study delves into the intricate interplay between global fossil fuel demands and the evolving landscape of the carbon market. Fossil fuels remain fundamental to global energy systems, yet the imperative to curb greenhouse gas emissions necessitates a comprehensive understanding of the dynamics shaping their consumption. Employing copula models, the research analyzes the tail dependence relationships between the carbon credits and four distinct energy sources including crude oil, coal, natural gas, and ethanol daily data. The study reveals that carbon-related emissions stemming from crude oil and coal demonstrate a pronounced reliance on carbon credits, whereas cleaner energy sources such as natural gas and ethanol exhibit a weaker correlation. Throughout the crisis period, there was a notable increase in the interdependence between the European Union Emissions Trading System (EU ETS) and most energy commodities, except for ethanol, which shows a decline in correlation. Notably, the relationship between EU ETS and natural gas appears insignificant. During market downturns, the observed low correlations offer beneficial diversification prospects. These findings underscore the need for nuanced risk management approaches and inform decision-making processes amidst the evolving dynamics of fossil fuel demands and carbon markets.

Abbreviations:

Energy Information Administration (EIA, 2019)
Emission Trading Schemes (ETS)
European Union Emissions Trading System (EU ETS)
Greenhouse Gas (GHG)
European Union (EU)
European Union Allowances (EUAs)
Brent crude oil (OIL)
Coal (COAL)
Natural gas (NGAS)
Ethanol (EtOH)
Global Financial Crisis (GFC)
European Climate Exchange (ECX)
United States of America (USA)
Akaike information criterion (AIC)

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Introduction:

The global demand for fossil fuels remains a pivotal aspect of the contemporary energy landscape, influencing economic development, geopolitical dynamics, and environmental sustainability. As societies grapple with the challenges posed by climate change, the role of fossil fuels and their correlation with the carbon market has garnered increasing attention. This exploration delves into the intricate relationship between global fossil fuel requirements and the functioning of the carbon market, elucidating the interplay, dependencies, and potential impacts on energy policies and market dynamics. Fossil fuels, despite advancements in renewable energy technologies, continue to serve as the primary energy source worldwide, powering industries, transportation, and households. Simultaneously, efforts to mitigate climate change have led to the establishment of mechanisms like carbon markets, aimed at reducing greenhouse gas

emissions by incentivizing emission reductions and fostering a transition to cleaner energy sources. Understanding the intersection between fossil fuel demands and the carbon market becomes crucial in navigating the complexities of sustainable energy transitions and climate action strategies on a global scale [1].

Global fossil fuel energy prices are predominantly governed by the interplay between demand and supply, dictating market trends. Market upswings typically lead to increased fuel costs, while downturns in demand tend to lower energy prices. Among consumable energy sources, oil holds primary status, followed by coal and natural gas as secondary sources. According to the Energy Information Administration (EIA, 2019), three primary sources contributed to 76% of total GHG emissions in 2018. Between 2011 and 2012, there was a substantial escalation in crude oil prices, reaching nearly 100 US dollars per barrel, followed by a gradual decline in subsequent years. Recent years have witnessed a notable surge in fossil fuel utilization, resulting in increased carbon emissions. Consequently, this surge has prompted the implementation of Emission Trading Schemes (ETS) aimed at mitigating carbon dioxide emissions and addressing the challenge of escalating global temperatures [2].

Throughout the last decade, coal prices followed a parallel trajectory to crude oil and accounted for approximately 40% of global power generation in 2018, according to the EIA, 2019. Countries are actively seeking to replace coal-fired power plants with eco-friendly energy solutions in the upcoming years, indicating a potential decline in coal usage. Conversely, natural gas prices show remarkable stability in the short term due to its acknowledged role as a more environmentally favorable alternative to coal [3]. The European Union Emissions Trading System (EU ETS) stands as the world's largest carbon trading market by capitalization, overseeing 43% of greenhouse gas emissions within the European Union (EU). With the involvement of over 11,000 firms across more than thirty-one economies, this system operates on a "Cap and Trade" structure [4]. Under this framework, participants face restrictions on their GHG emissions.

Enterprises receive carbon allowances, known as European Union Allowances (EUAs), based on their power plant capacities. When companies emit fewer greenhouse gases than their permissible limit, surplus EUAs are traded within the market. Conversely, entities surpassing the emission limits engage in purchasing these allowances. This mechanism characterizes the operational functioning of the European carbon market [5]. The EU ETS operates through distinct phases, each imposing specific compliance requirements. Phase I, a trial period, spanned from January 1, 2005, to December 31, 2005, followed by Phase II from 2006 to 2012. Phase III covered the years 2013 to 2020. Currently, the EU ETS operates in its fourth phase, known as Phase-IV, where an annual reduction of the EU cap is set at 1.74%.

Initially, Phase I's generous allocation of permits resulted in significant price fluctuations, driving valuations nearly to zero. Consequently, the trend of freely distributing carbon allowances is gradually shifting toward auctions, especially within the power sector. Carbon assets now form a vital aspect of production, causing carbon price fluctuations to intertwine intricately with the behavior of other energy commodity markets [6]. In recent times, the carbon market has displayed notable volatility characterized by significant fluctuations and a leptokurtic distribution in volatility. Additionally, various studies point to a strong dependence on traditional energy commodities [7]. Economic downturns often lead to reduced production output among EUA participants, resulting in an excess of unused allowances. This surplus supply of EUAs tends to drive down their market pricing, suggesting a negative correlation between carbon costs and energy prices.

Conversely, during economic upswings, increased consumption and production among EUA participants from the European Union and the United States are expected. This scenario is likely to elevate costs for energy commodities and lead to higher carbon dioxide (CO₂) emissions, implying a positive correlation between EUAs and energy prices. These

interconnections highlight the potential for diversification advantages [8]. Initially, an analysis was conducted to explore the interdependence between EUAs and crude oil by utilizing copulas. Subsequently, studies were carried out to examine the dynamic relationship and volatility spillover among fossil energy sources including oil, gas, coal, and electricity, and European carbon pricing. These investigations revealed a significant linkage and spillover effects between the variables, highlighting the influence of energy prices on the dynamics of the CO₂ emission market. Our research findings align, to some extent, with recent studies that explore tail dependence patterns observed in carbon markets and energy commodities [9].

This study contributes uniquely to existing literature in three significant ways. Unlike previous research that predominantly concentrates on oil, natural gas, coal, and electricity, this study extends its analysis to include crude oil and coal as high carbon-emission fuels, while considering natural gas and ethanol as lower carbon-emission fuels. By categorizing these fuels, a comprehensive understanding of their interdependence during market fluctuations is attained [10]. The study utilizes Copula methodologies to evaluate interdependence and tail interdependence among markets. Specifically, seven distinct bivariate copulas are employed to illustrate the relationship between EUA and energy variables. The selection of the appropriate copula is based on minimizing the Akaike Information Criterion (AIC) [11] [12].

The outcomes of this study offer valuable guidance for investors aiming to diversify their portfolios and proficiently manage their assets. Moreover, policymakers can leverage these findings as a resource to shape energy policies focused on efficiently controlling greenhouse gas emissions. The study's data period spans the oil price decline between 2014 and 2018. This inclusive timeframe enables an examination of how the market's dependence on external factors reacts to substantial financial fluctuations [12].

Literature Review:

This section aims to examine existing scholarly works to establish context and relevance. The EU ETS stands out as the largest and most expansive global carbon market. Governed by the ETS, this system grants enterprises the authorization to emit carbon dioxide in return for allocated carbon allowances, also known as carbon credits. An allowance typically represents the authorization to emit one metric ton of CO₂. As per the EU ETS Directive, the transfer of allowances adheres to the conditions outlined in the directives of EUA 2019. It's important to note that the definitions and operational mechanisms of carbon assets exhibit variability [13] [14]. The asset held by enterprises for greenhouse gas generation has been acknowledged. Conversely, it can be categorized as public finance, entailing financial compensation provided by wealthier nations to developing countries in response to environmental pollution, as discussed [15].

Various factors influence carbon pricing. One study analyzed the macroeconomic aspects of European Union Allowance pricing. The findings indicated a limited correlation between carbon market factors and stock-bond variables. However, an alternative analysis revealed strong correlations among stock prices, European Union Allowances (EUAs), and industrial production levels. Additionally, economic recessions were observed to negatively impact carbon prices. Another research, led by [16], showcased a significant correlation between exchange rates and carbon prices. The authors argued that this relationship primarily stems from the energy switching mechanism, as exchange rates influence the prices of carbon, natural gas, and coal to varying extents.

Several scholarly investigations have focused on exploring the relationship between European allowance prices and electricity prices [17]. These studies operate under the premise that carbon prices significantly influence the electricity sector due to its substantial contribution to total CO₂ emissions in the European Union. For instance, electricity costs are intricately tied to both fuel prices and EUA rates. Furthermore, fluctuations in electricity prices can also impact

their variability. Additionally, prior studies have shown that fluctuations in EUA prices are influenced by meteorological conditions [18].

The empirical exploration into the relationship between the carbon market and energy commodities initially centered on examining the linkages between carbon and energy. These studies highlighted a correlation between carbon pricing within the EU ETS and the prices of crude oil, natural gas, and coal. In furtherance of these studies, subsequent research delved into analyzing the connection between the costs related to EUAs and electricity prices. Additionally, this research investigated how natural gas prices impact both EUA and electricity prices [19]. To accomplish this, they utilized a structural, co-integrated vector-error-correction model. A study revealed evidence indicating a causal relationship between fluctuations in coal and natural gas prices and the EUA futures. This study specifically focused on investigating the non-linear correlation between carbon and energy markets [20].

Multiple scholarly investigations have utilized copula models to evaluate the interdependence between EUA and oil prices. Copula techniques enable the quantification of marginal distributions, whereas simple correlation coefficients offer insights into the dependency degree. This study opts for basic bivariate copulas due to the complexity involved in constructing high-dimensional copulas. The rationale is that employing multiple copulas becomes essential to ascertain the most suitable fit, given the diverse levels of interdependence across different markets. Thus, this research selects seven copulas for analysis purposes [21].

The third aspect to consider in this study revolves around its focus on the utilized data and methodologies. The initial facet involves the collection and analysis of data, encompassing an examination of the potential price movements of EUA, Brent crude oil, coal, natural gas, and ethanol. EUAs futures contracts are actively traded on the European Climate Exchange (ECX), denominated in Euros per metric ton [22][23]. These energy commodities all have negative environmental impacts, with oil and coal being primary contributors to carbon emissions, while natural gas and ethanol exhibit relatively lower emissions.

The study sources data from DataStream, a comprehensive global database provided by Thomson Reuters, spanning from January 5, 2009, to May 31, 2021. This timeframe corresponds to a period marked by the restoration of market stability after the Global Financial Crisis (GFC). The selection is influenced by the extensive coverage of the crisis in research conducted by various scholars. The study primarily focuses on the contemporary energy dilemma [24].

The price trends in the carbon market and energy commodities illustrate distinct patterns. The United States of America (USA) witnessed a consistent downward trend from mid-2011, possibly influenced by the European debt crisis. However, there's a noticeable surge in data between 2017 and 2022, likely attributed to the economic rebound after the oil price crisis. There has been a significant increase in energy prices globally, accompanied by rising production and consumption trends. Crude oil and coal prices show comparable trends, while natural gas and ethanol demonstrate a consistent positive correlation, except for a period between 2016 and 2019.

The statistical summary of the logarithmic return series for EUA and energy commodities reveals distinctive patterns. EUA exhibits the largest standard deviation, followed by NGAS, OIL, EtOH, and COAL. Additionally, the mean value is negative for NGAS and positive for all other variables [25].

Methodology:

Data Collection:

Daily data on the European Union Allowance (EUA) and four distinct energy sources including crude oil, coal, natural gas, and ethanol were collected for the specified analysis period.

Copula Modeling:

Copula models were employed to analyze the tail dependence relationships between EUA and the identified energy commodities. Copula functions enable the exploration of

complex dependencies, specifically capturing the joint distribution of variables and their extreme movements. The statistical significance of mean and variance coefficients is presented, indicating significance at a maximum level of 10%.

Correlation Analysis:

Statistical analysis was conducted to assess the strength and nature of correlations between EUA and each energy source. Examination of correlation trends over different time periods, including crisis periods and market downturns, was performed.

Identification of Trends and Patterns:

Identification and analysis of trends in the interdependencies between EUA and energy commodities, particularly focusing on shifts during crisis periods and market downturns.

Risk Implications and Diversification Strategies:

Assessment of implications for risk management among investors and policymakers engaged in emission trading regulations based on the observed correlations. Evaluation of the potential for diversification strategies, considering the findings of significant and insignificant correlations during various market conditions.

Validation and Sensitivity Analysis:

Sensitivity analysis was conducted to validate the robustness of the findings and assess the stability of the observed relationships under different scenarios or variations in the dataset.

Interpretation and Conclusion:

Interpretation of results and conclusions drawn from the analysis to provide insights into the dynamics between fossil fuel demands, carbon markets, and the implications for risk mitigation strategies [26].

Results and Discussion:

The outcomes of the Bivariate Copula model for the entire dataset are detailed. The estimation results of seven copulas for the entire sample period are provided, where the copula with the lowest Akaike Information Criterion is selected for each pair. The level of dependence is observed to be highest between the European Union Allowances-Oil pair across most copulas, while the European Union Allowances-Natural Gas pair exhibits the lowest dependence level. Notably, the Gaussian (Normal) copula demonstrates the best fit among all copulas and variable pairings, evidenced by its lowest Akaike Information Criterion for the European Union Allowances-Oil and European Union Allowances-Ethanol pair. Conversely, the Student-t and Clayton copulas are found suitable for European Union Allowances-Coal and European Union Allowances-Natural Gas, respectively.

The dataset is divided into two regimes: Regime 1 spans from March 2012 to June 2020, while Regime 2 specifically encompasses the crisis period from April 2015 to June 2017, characterized by the decline in oil prices since 2015. The copulas that exhibit the best fit for the entire sample are employed to assess fluctuations in interdependence and correlation intensity during the crisis. The findings reveal an upward trend in reliance and Kendall's tau correlation from Regime 1 to Regime 2 for most pairs, except for EUA-Ethanol, where it decreases. Thus, distinct commodity reactions are evident during crises.

Previous research, such as Benz and Truck's study in 2006, has demonstrated the effectiveness of emission trading programs in reducing environmental harm. While past research mainly focused on basic correlations between carbon and energy commodities, our contribution lies in utilizing seven distinct bivariate copulas. This study primarily centers on EU-ETS and its relationship with other energy factors, showcasing a strong reliance of the carbon market on oil and coal due to their higher pollution levels as energy sources.

These findings align with existing research. However, the lower reliance on EUA and natural gas, as well as ethanol, echoes similar findings from previous studies. The limited interdependence observed between the carbon market and natural gas and ethanol suggests a relatively lower environmental impact compared to oil and coal. A marginal model is a necessary

condition for conducting copula estimations. This study employs the ARMA-GARCH Student-t model and presents the findings based on lag values ranging from zero to one. The selection of the most appropriate model is determined by identifying the model with the lowest AIC value. Multiple bivariate copulas are employed in order to capture the tail dependence structure between the carbon market and traditional energy commodities. The statistical significance of the mean and variance coefficients indicates that the majority of these coefficients are significant at a maximum significance level of 10%. The Ljung-Box values do not provide sufficient evidence to reject the null hypothesis that there are no ARCH effects and the data exhibits homoscedasticity. Therefore, these tests demonstrate the model's adequacy and adherence to the specified criteria.

The estimation results of seven copulas for the entire sample period are showcased. The copula that best fits each pair is determined by selecting the one with the lowest AIC. The level of dependence is shown to be highest between the European Union Allowances-Oil pair across most copulas, while the European Union Allowances-Natural Gas pair exhibits the lowest level of dependence. The Gaussian (Normal) copula demonstrates the most optimal fit among all copulas and pairings of variables, as evidenced by its lowest AICs for European Union Allowances-Oil and European Union Allowances-Ethanol. Conversely, the Student-t and Clayton copulas are found to be suitable fits for European Union Allowances-Coal and European Union Allowances-Natural Gas, respectively. The Kendall's tau correlation is statistically significant for all pairs, with the exception of the European Union Allowances-Natural Gas pair. This finding substantiates the lack of a substantial correlation between the carbon market and natural gas. Table 1 showcases the copula that best fits each pair based on the lowest AIC, along with the level of dependence and the significance of Kendall's tau correlation for each pair. The copula chosen for each pair and their respective characteristics are highlighted.

Table 1: Estimation Results of Copulas, Dependence Levels, and Kendall's Tau Correlation for EUA and Energy Commodities

Pair	Copula	AIC	Level of Dependence	Kendall's Tau Correlation
EUA-Oil	Gaussian	Lowest	Highest	Significant
EUA-Natural Gas	Clayton	Lowest	Lowest	Not Significant
EUA-Ethanol	Gaussian	Lowest	Intermediate	Significant
EUA-Coal	Student-t	Lowest	Intermediate	Significant

Table 2: Descriptive Tests for Coal, Oil, Gas, and Ethanol

	Carbon	Oil	Coal	Gas	Ethanol
Mean	0.0055	-0.0284	-0.0302	-0.0438	-0.0068
Maximum	25.5254	18.0774	9.4236	35.4507	6.7058
Minimum	-42.1441	-28.9762	-11.8235	-16.0909	-9.7934
Std Dev	3.2280	2.5231	1.6759	2.8820	1.2832
Skewness	-0.9124	-0.8055	-0.5799	2.0116	-0.2313
Kurtosis	14.8020	14.3438	4.7712	22.7794	5.3785

The dataset is categorized into two regimes. Regime 1 encompasses the entire data range, spanning from March 2012 to June 2020, while Regime 2 specifically encompasses the crisis period from April 2015 to June 2017. Regime 2, on the other hand, pertains specifically to a period commencing from April 2015 to June 2017. The period is characterized by significant events such as the decline in oil prices since 2015. The copulas that exhibit the highest degree of match to the whole sample are employed to assess fluctuations in interdependence and the intensity of correlation during these periods. The findings reveal that the level of reliance and Kendall's tau correlation exhibit an upward trend from Regime 1 to Regime 2 for all pairs, with

the exception of European Union Allowances-Ethanol, where it demonstrates a decrease. Therefore, it is evident that during times of change, various commodities exhibit distinct reactions. Table 2 presents Descriptive test statistics for European Union Allowances, Crude Oil, Coal, Natural Gas, and Ethanol.

Table 3: Copula Model Estimations for EUA and Energy Commodities

Model	EUA/Oil	EUA/Coal	EUA/Gas	EUA/Ethanol
P	0.195***	0.17***	0.009	0.14***
S.E	0.016	0.017	0.02	0.02
LL	62.55	48.79	0.33	31.44
Copula Type	AIC	BIC	Kendal τ	
Normal Copula	-123.11	-117.02	0.13***	
Student-Copula	-123.11	-117.02	0.13***	
Clayton	-106.19	-100.11	0.10***	
Gumbel	-88.22	-82.14	0.10***	
Frank	-110.30	-104.22	0.12***	
Joe	-52.73	-46.65	0.06***	

Note: The values presented are estimates of Copula models depicting the relationship between EUA and respective energy commodities (Oil, Coal, Gas, Ethanol). The statistical significance levels are denoted as *** for p-values less than 0.001, ** for p-values less than 0.01, and * for p-values less than 0.05.

Table 4: Copula Model Estimations for EUA and Energy Commodities during Complete Sample and Crisis Period

Model	EUA/Oil	EUA/Ethanol	EUA/Coal	EUA/Gas
ρ	0.195***	0.22***	0.18***	-
S.E	0.016	0.02	0.018	0.018
LL	62.55	46.45	53.94	1.12
Copula Type	AIC	BIC	Kendal τ	
Normal copula	-123.11	-117.02	0.13***	-
Student copula	-90.89	-85.38	0.12***	-
Clayton	-	-	-	0.01*

Table 4 showcases estimations of Copula models for EUA and energy commodities (Oil, Coal, Gas, Ethanol) during the complete sample and crisis period. The table provides correlation coefficients, standard errors, log-likelihood, AIC, Bayesian Information Criterion, and Kendall's τ values for various Copula models utilized in the analysis. Emission trading programs have been found to be effective in reducing environmental harm, as demonstrated by the research conducted by Benz and Truck in 2006. Although previous research has mostly focused on examining the basic correlations between carbon and energy commodities, there has been a lack of investigation into the tail dependency dynamics of these markets. Our contribution to the existing body of literature is in the utilization of seven distinct bivariate copulas. Our research primarily centers around the EU-ETS and its relationship with other energy factors. The carbon market exhibits a pronounced reliance on oil and coal due to their higher levels of pollution as energy sources. These findings are consistent with the research conducted. Nevertheless, the correlation between EUA and natural gas as well as ethanol exhibits a lower degree of reliance, aligning with the findings of the limited interdependence observed between the carbon market and natural gas and ethanol indicates that these energy sources have relatively lower environmental impact compared to oil and coal.

Discussion:

This study delves into the complex interplay between global fossil fuel demands and the evolving landscape of the carbon market, utilizing copula models to explore their relationships. The findings highlight significant connections between emissions from crude oil and coal and EUA carbon credits, indicating a substantial reliance on the carbon market dynamics for these sources. Conversely, cleaner energy sources like natural gas and ethanol exhibit weaker correlations, suggesting a lesser dependency on carbon credits. During periods of crisis, the study reveals shifts in interdependence: while most energy commodities demonstrate an increased correlation with the EU ETS, ethanol displays a decrease in correlation. Notably, the relationship between EUA ETS and natural gas appears insignificant during these turbulent periods. The observed low correlations during market downturns offer promising diversification opportunities, particularly beneficial for risk management strategies. These findings underscore the need for refined risk management approaches, emphasizing the nuanced dynamics influencing different energy sources' reliance on the carbon market. Ultimately, this nuanced understanding of the relationship between fossil fuel consumption and the carbon market carries significant implications for decision-making. It underscores the necessity for informed strategies that acknowledge varying dependencies among energy sources. This insight is invaluable for stakeholders and policymakers navigating the complex terrain of energy markets while addressing pressing environmental concerns.

Conclusion:

Fossil fuel use raises environmental concerns that can lead to social, economic, and health challenges. To address this, emission reduction systems are being implemented. Carbon pricing mechanisms are seen as a way to reduce dependence on traditional energy sources. This study aims to contribute insights into diversification by examining how the European Union relies more on oil and coal than ethanol and natural gas, especially during crises. The carbon market appears to offer opportunities for diversifying energy investments. These findings are crucial for investors, policymakers, and portfolio managers, suggesting the need for diverse portfolios and improved risk management strategies. While this study focuses on the carbon and energy market, future research could expand to include renewable energy, stock, and metal markets. Additionally, exploring the relationship between the European and Chinese carbon markets and using advanced copula methodologies could provide a more comprehensive view of these markets.

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